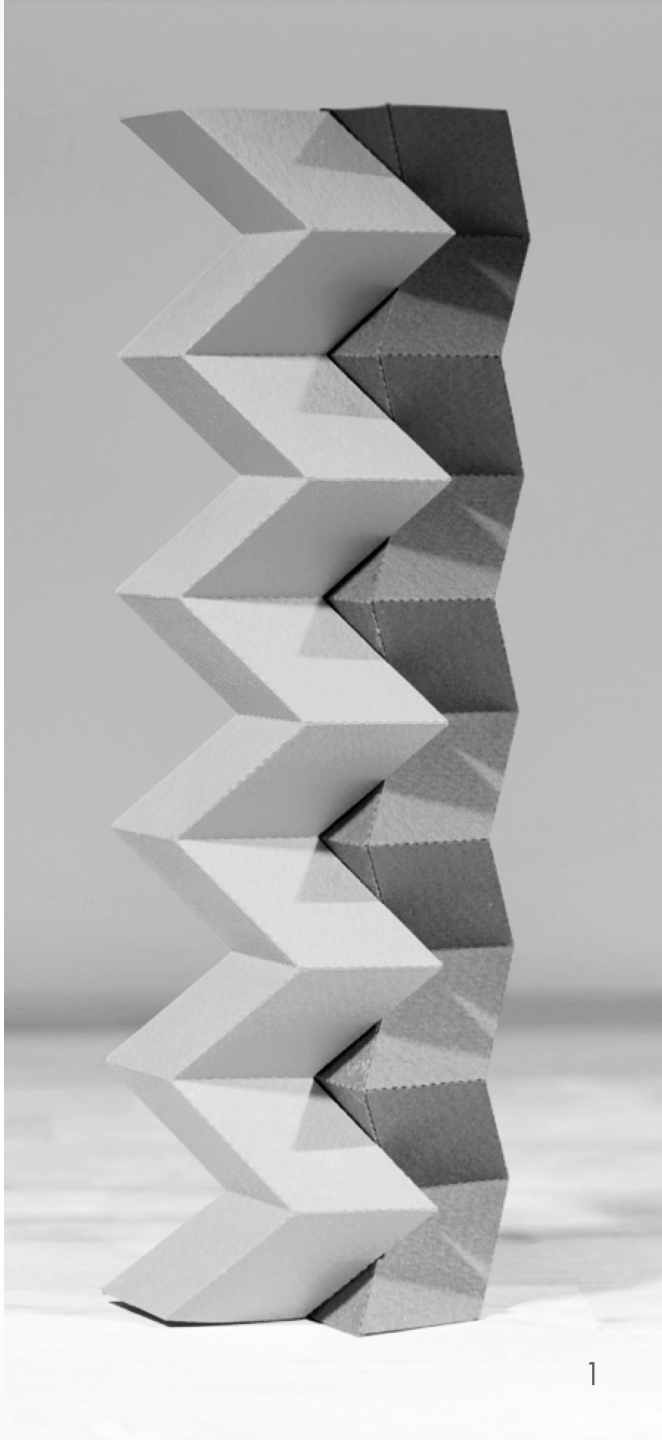
Department of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign
October 28, 2016

Tailoring the Stiffness of Deployable Origami Structures

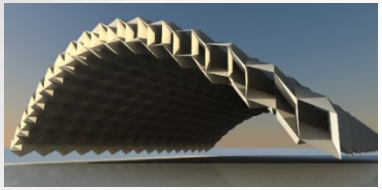


PhD Student : Evgueni Filipov

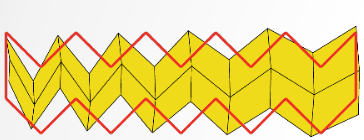
Committee:

Prof. Glaucio H. Paulino (Chair)
Prof. Paolo Gardoni
Prof. Arif Masud
Dr. Mark Schenk
Prof. Tomohiro Tachi

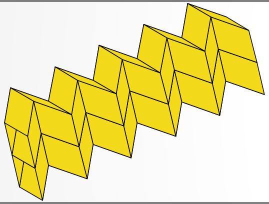
Presentation Outline



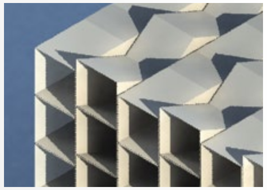
Origami in Engineering



Structural Analysis of Origami

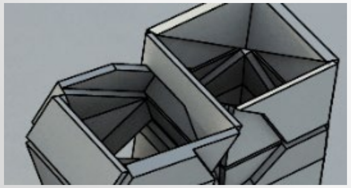


Coupling of Origami Tubes



Cellular Assemblages and Coupled Variations

Tubes with Polygonal Cross-Sections



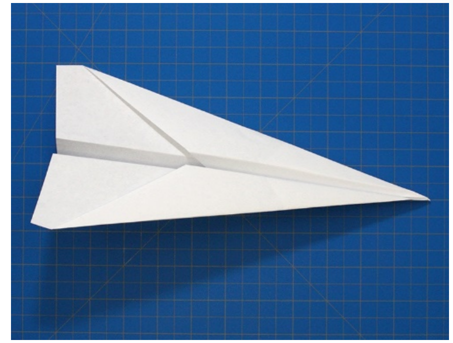
Concluding Remarks

Origami as Art

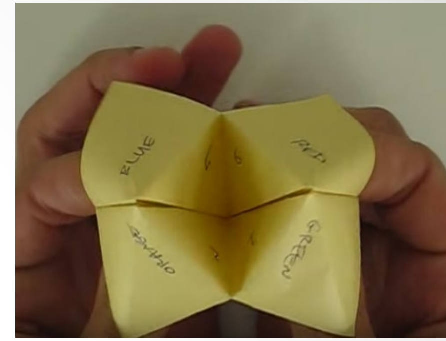
Orchfini by E. Joisel
www.ericjoisel.com/gallery.html

Elk 358 by R. Lang
http://www.langorigami.com/

Origami as Entertainment



Paper Airplane
http://www.foldnfly.com/



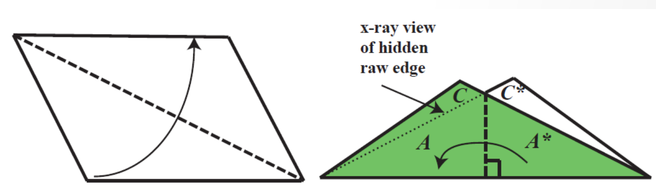
Origami Fortune Teller
www.origami-instructions.com/

Origami as Fashion

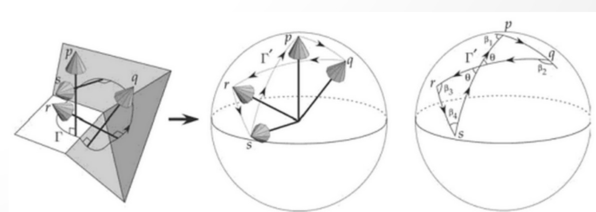
Origami Dress
http://www.julewaibel.com/

Origami Bracelet
www.tinederuysser.com/

Origami in Education



Geometry - A. Tubis *6OSME* 2014



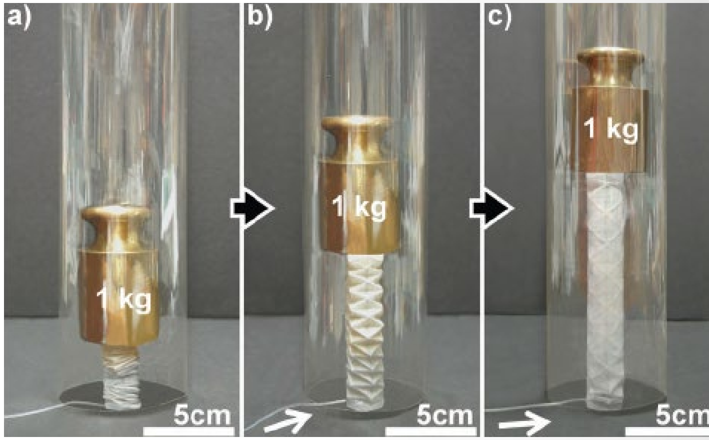
Gaussian Curvature - T. Hull 2012

Engineering Applications of Origami

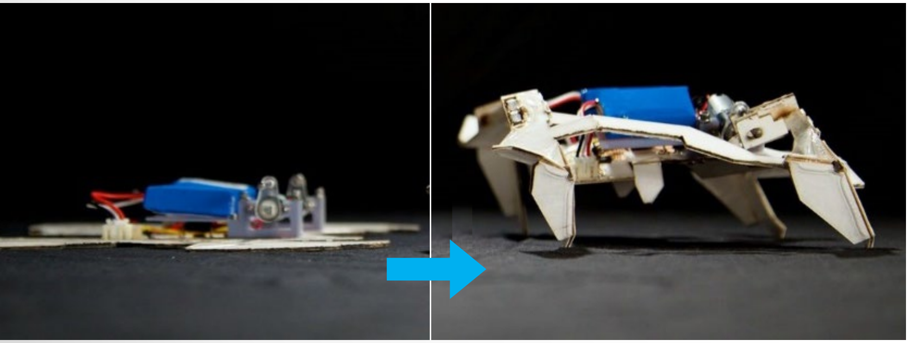
- Compact
- Deployable
- Pre-Fabricated
- Self-Assembly
- Tunable
- Multi-Functional
- Adaptable

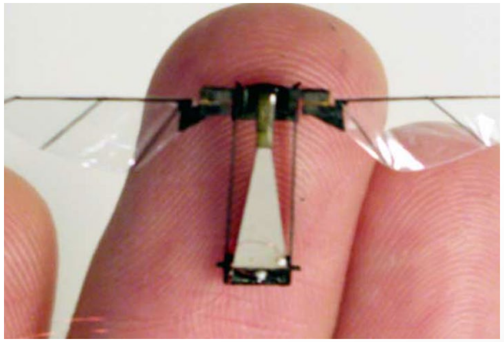

Kiefer Technic Showroom

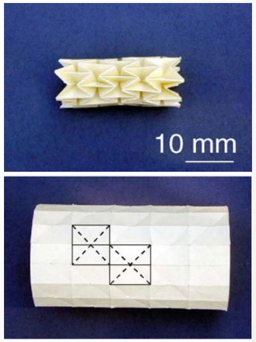

ISS – NASA 2011



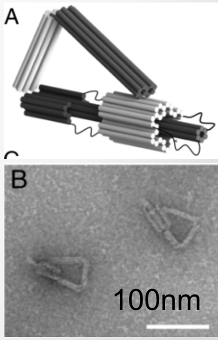
Martinez et al. (2012)


Felton et al. (2014)


Wood (2008)

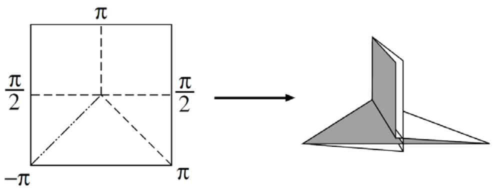


Kuribayashi et al. (2006)



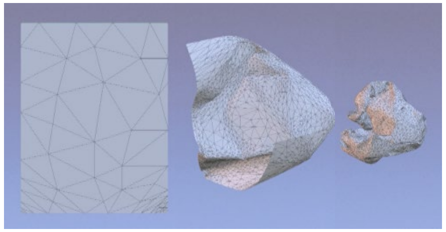
Marras et al. (2015)

Theory and Analysis

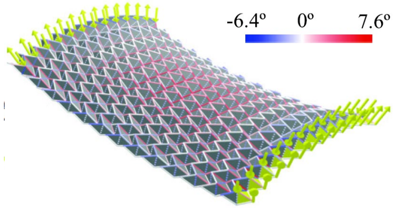


Belcastro and Hull (2013)

Demaine and Demaine (2012)

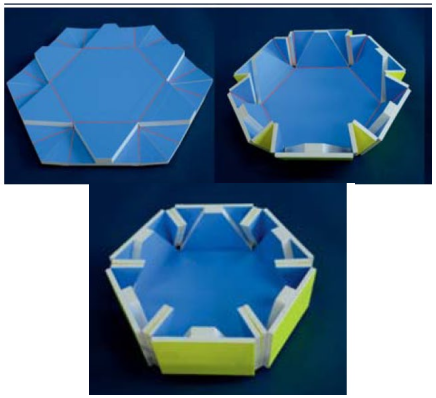


Narain et al. (2013)

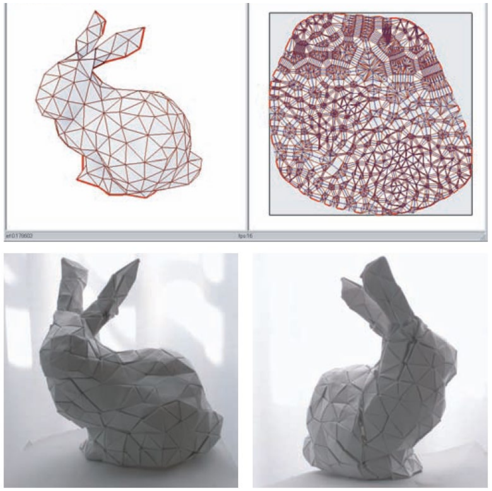


Wei et al. (2013)

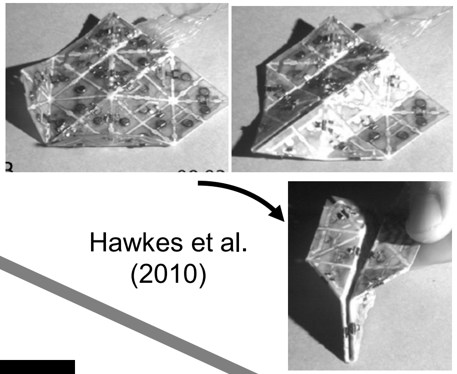
System Design



Chen et al. (2015)



Tachi (2010)



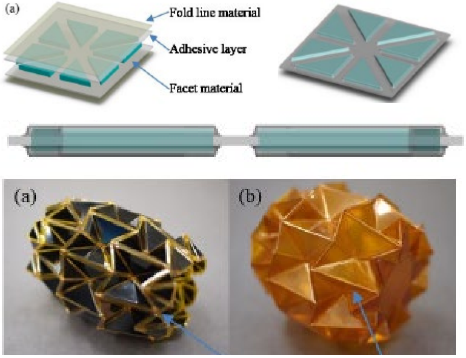
Hawkes et al. (2010)

Origami Engineering

Materials & Fabrication

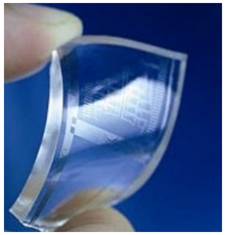
Black LAB Architects (2014)

C. Hoberman (2012)



Lee et al. (2013)

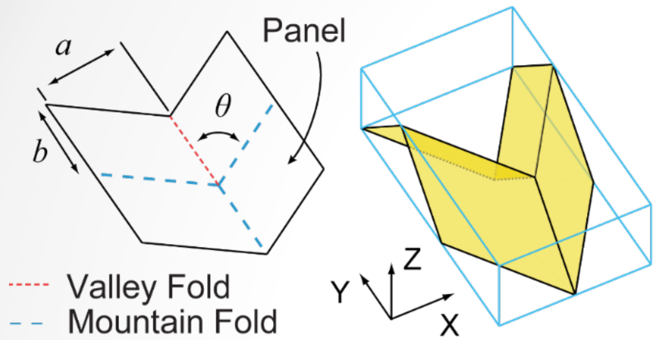
Living hinge

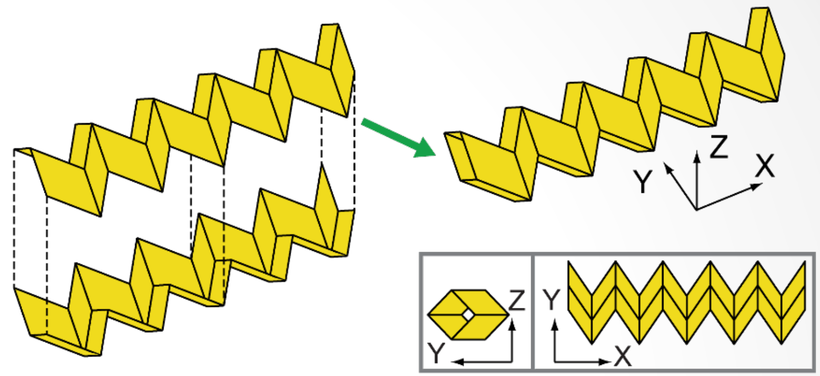


Graphene sheet

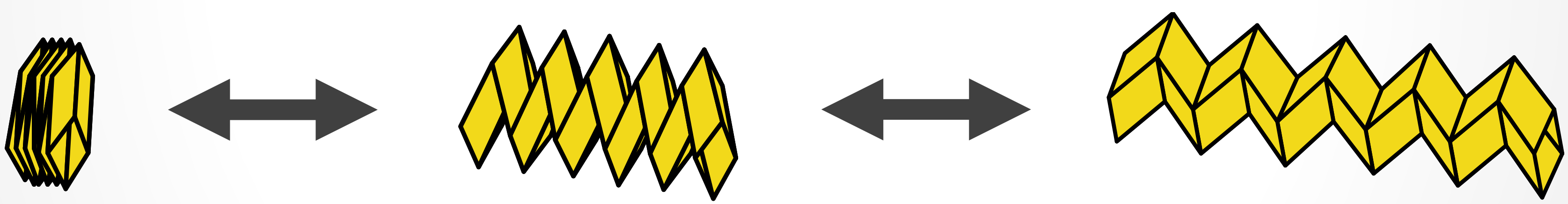
Miura-ori Tube Origami

Miura-ori → Tube

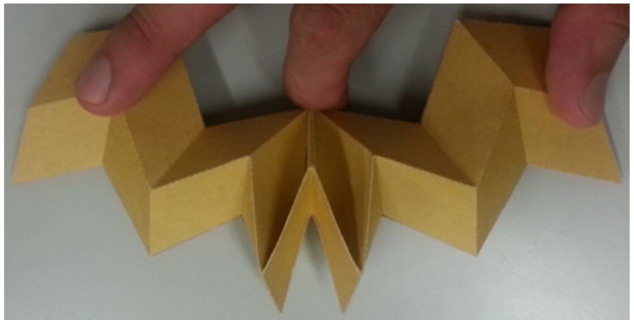




Kinematic “rigid” folding



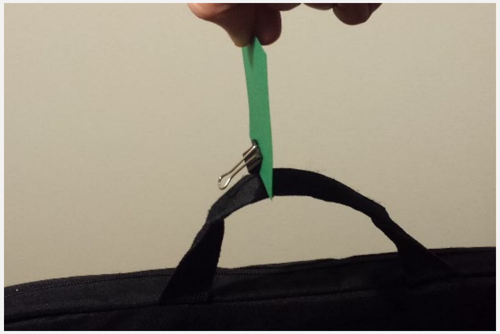
Elastic deformations



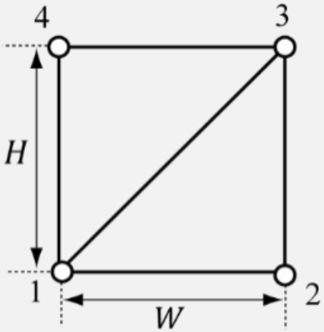
Elastic Modeling for Origami

S

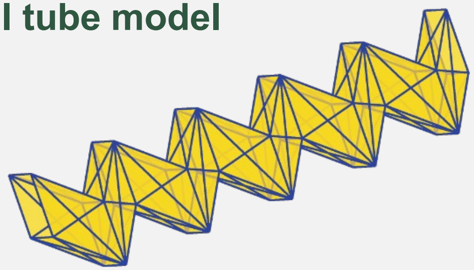
Panel Shear & Stretching



Model with bars elements



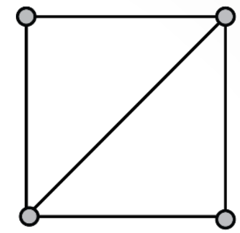
Full tube model

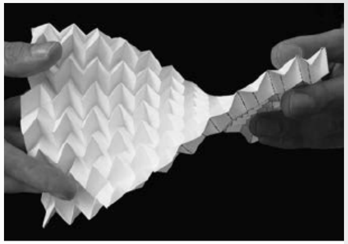


Evolution of the *Bar and Hinge* Model

Schenk and Guest (2010)

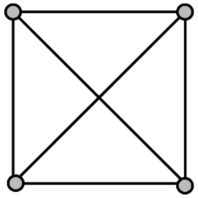
1. **Simplicity in the design and use**
2. **Insight on stiffness properties**

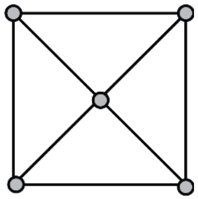
1 
N4B5

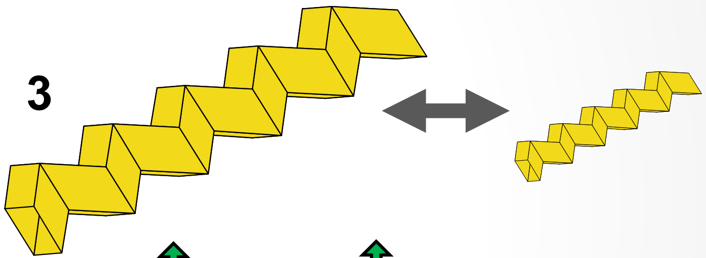
2 

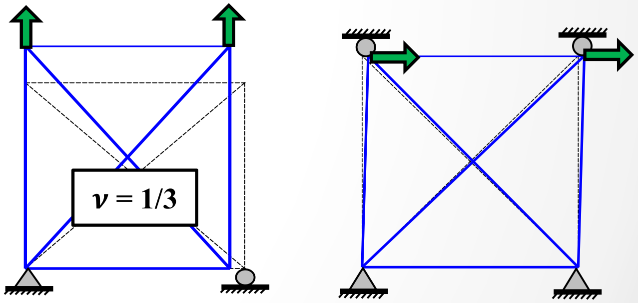
Filipov, Tachi, Paulino (2016) *Origami 6*

3. **Scalability**
4. **Model isotropy**
5. **Material properties**
   - Thickness t
   - Poisson's Ratio ν
   - Young's modulus E
   - Density ρ


N4B6

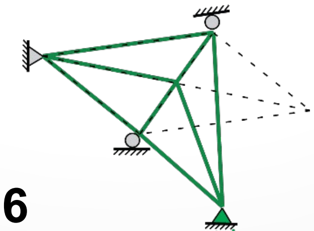

N5B8

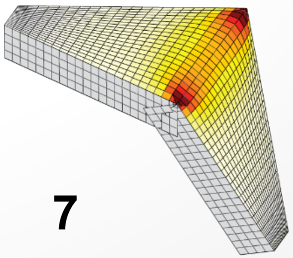
3 

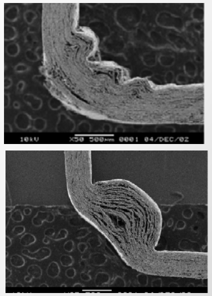
4, 5 

Filipov, Liu, Tachi, Paulino (2016) In Preparation

6. **Large displacements**
7. **Geometric influence**
8. **Fold line stiffness**

6 

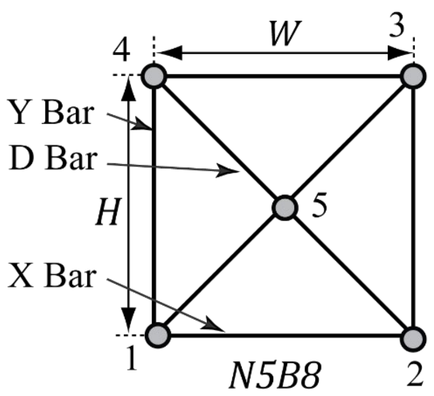
7 

8 

Nagasawa et al. (2003)

S Bar Model for Panel Stretching & Shear

Bar stiffness definitions

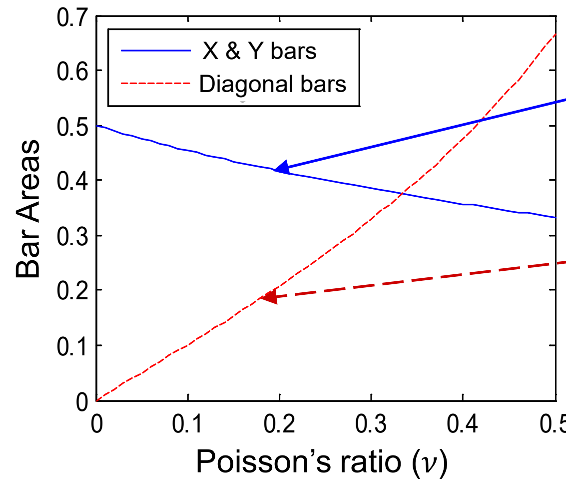


$$K_S = \frac{EA_{Bars}}{L}$$

Young's modulus: E

Thickness: t

Poisson's ratio: ν



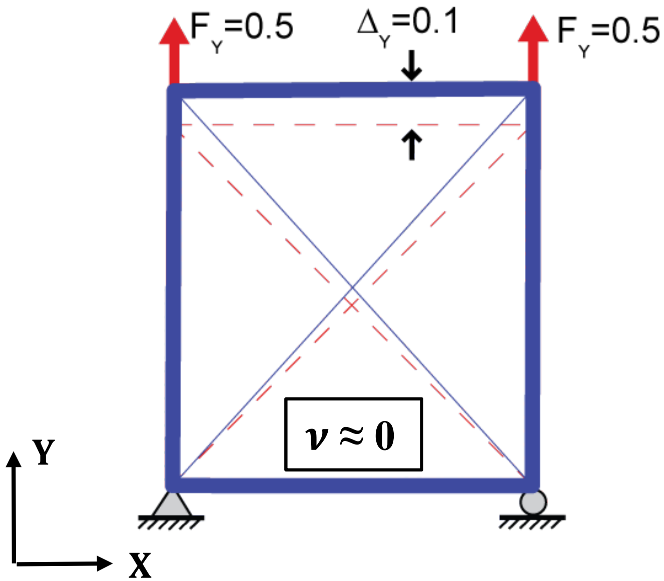
Bar areas:

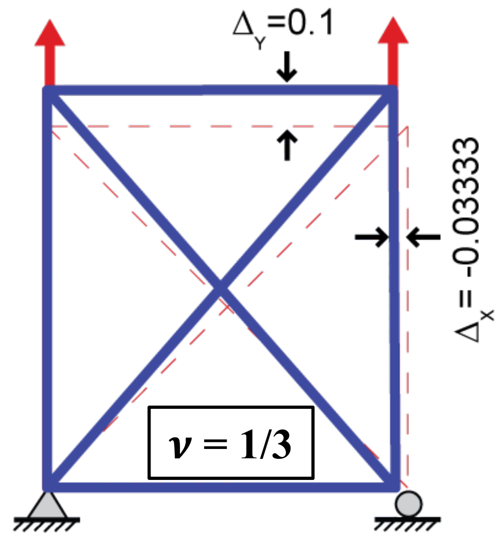
$$A_Y = t\,\frac{W^2 - \nu H^2}{2W(1-\nu^2)}$$

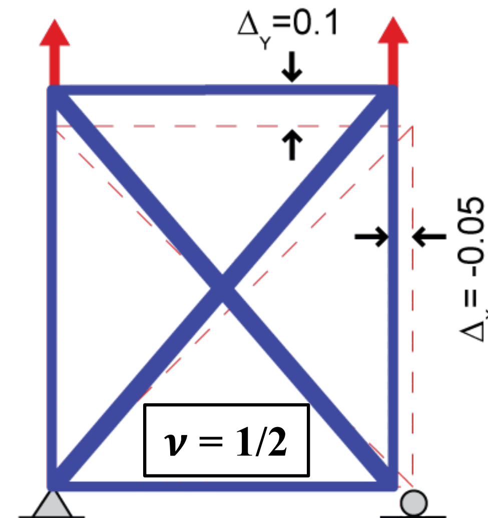
$$A_D = t\,\frac{\nu(H^2 - W^2)^{3/2}}{2HW\,(1-\nu^2)}$$

Poisson effect

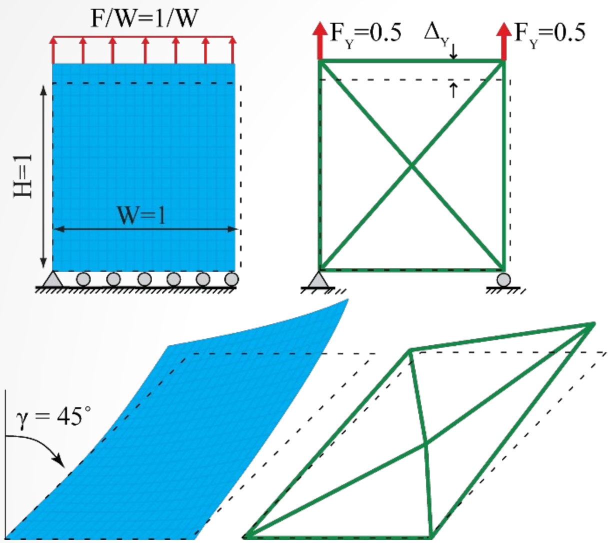
$$\varepsilon_x = \nu * \varepsilon_y$$

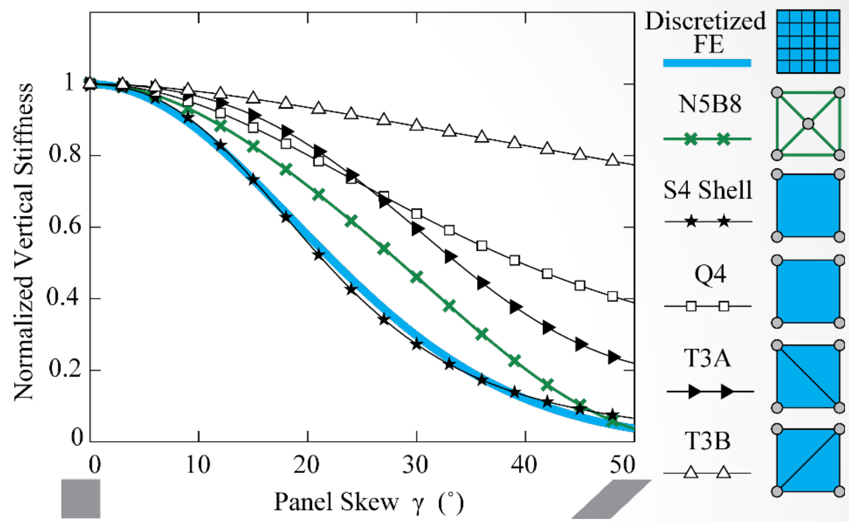


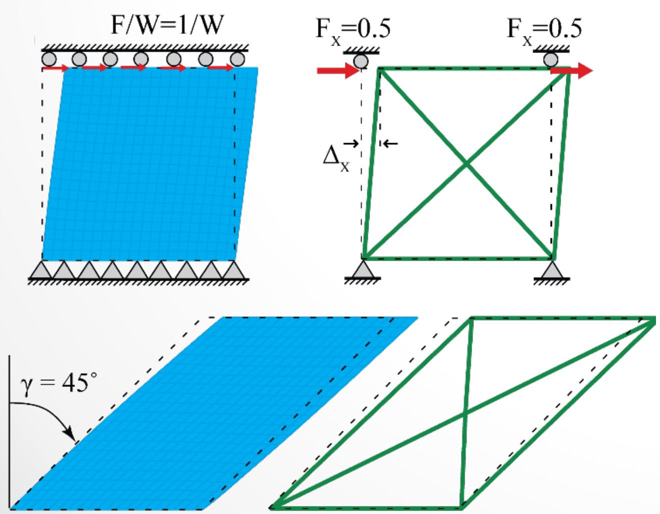


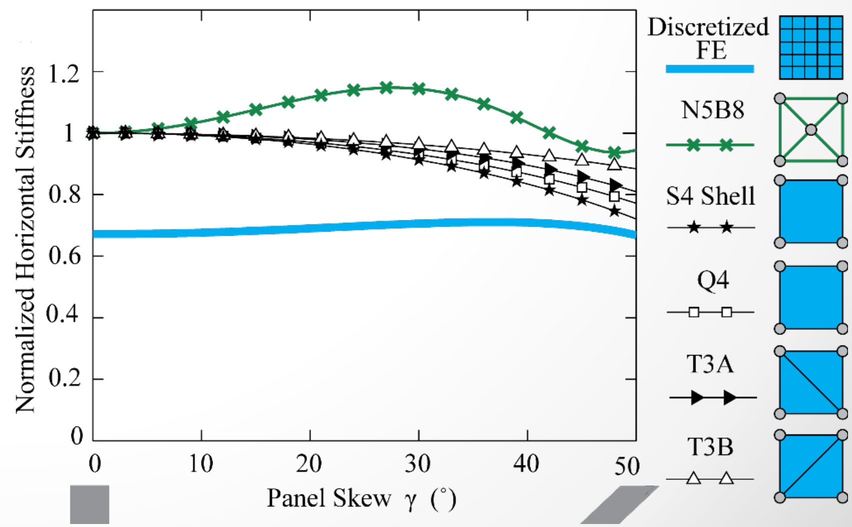


S Stiffness of Panel Stretching & Shear



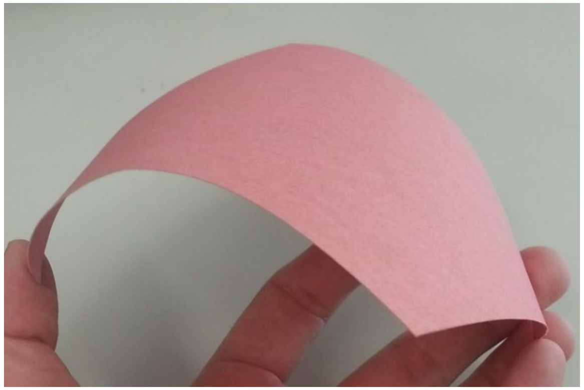




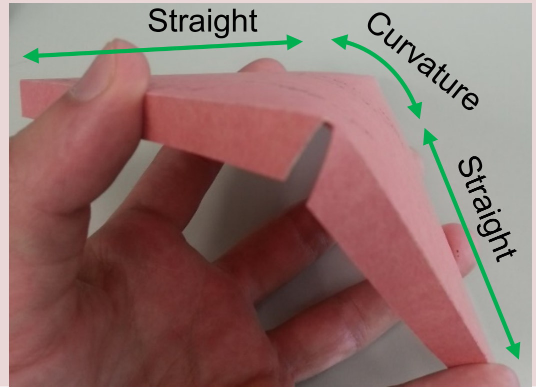


B Bending Thin Sheet with Restricted Edges

Constant curvature bending

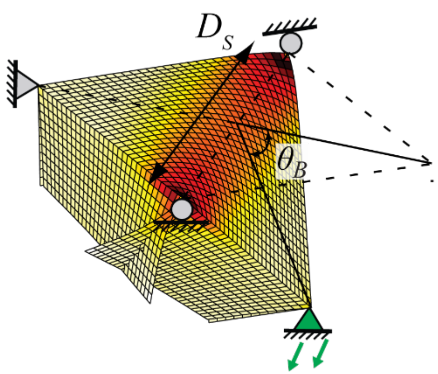


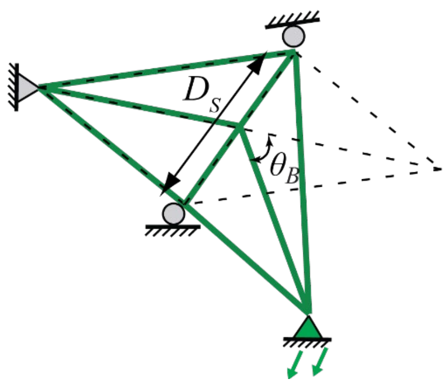
Bending restricted at edges



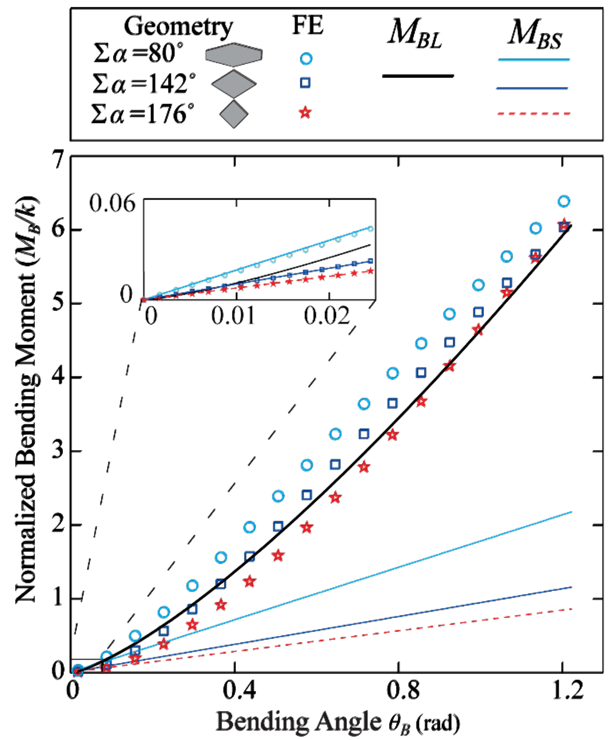
- Bending localized in short diagonal
- Stiffness is higher than with constant curvature bending

Lobkovsky AE (1996)

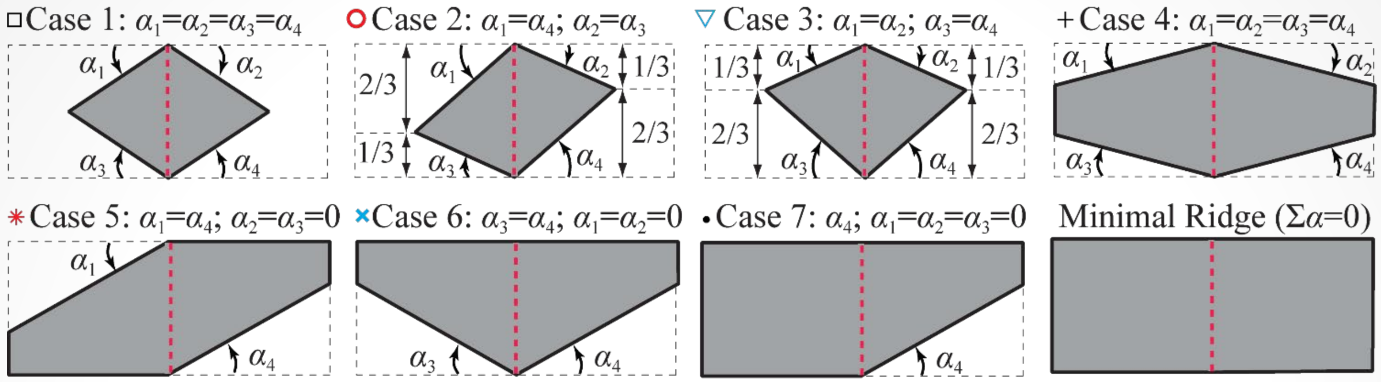




$$M_{BL} = \theta_B{}^{4/3}(1.0)\, k \left(\frac{D_S}{t}\right)^{1/3} \qquad k = \frac{Et^3}{12(1-\nu^2)}$$

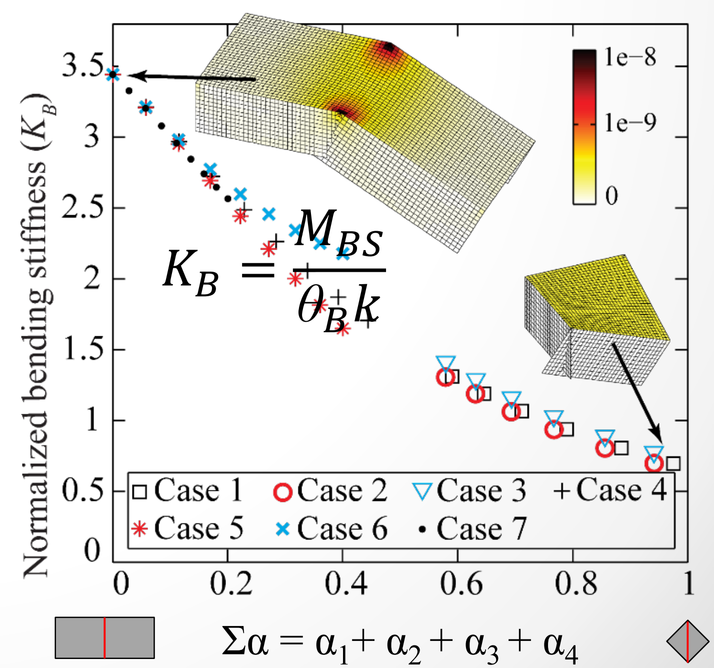


B Geometric Influence on Bending Stiffness



Small displacement bending (θ_B = 1°)

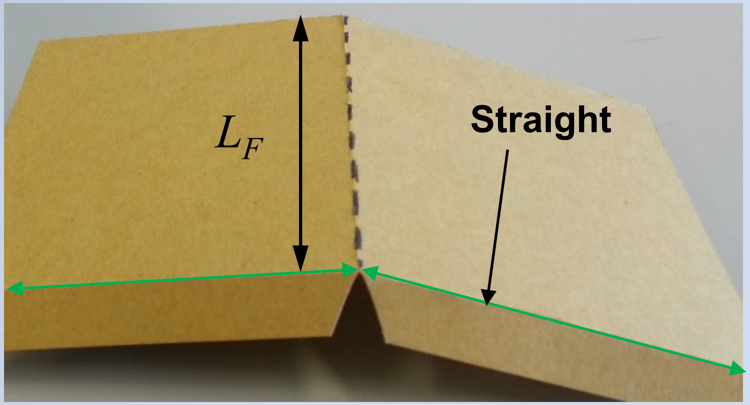
$$M_{BS} = \theta_B \left(0.55 - 0.42\frac{\Sigma\alpha}{\pi}\right) k \left(\frac{D_S}{t}\right)^{1/3}$$



$$K_{BS} = \left(0.55 - 0.42\frac{\Sigma\alpha}{\pi}\right) k \left(\frac{D_S}{t}\right)^{1/3}$$

F Local Stiffness of Prescribed Folds

Bending at prescribed fold line



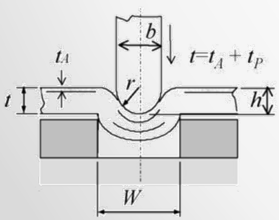
- Stiffness scales with length L_F and k

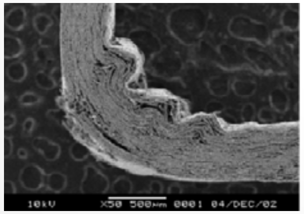
$$K_{FL} = \frac{L_F}{L^*} k$$

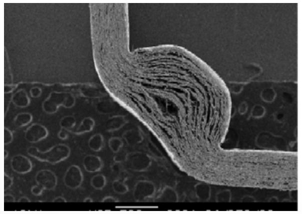
- L^* is a length scale factor
- L^* may depend on physical and material properties

Lechenault et al. (2014)

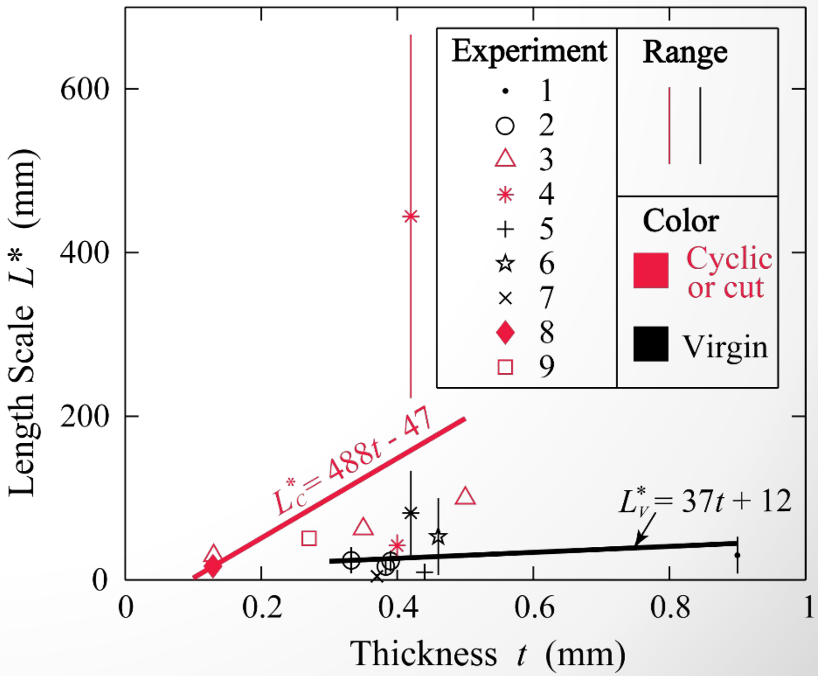
- ■ Experiments where sample is cut, perforated or cycled
- ■ Virgin loading of pre-creased paperboard samples







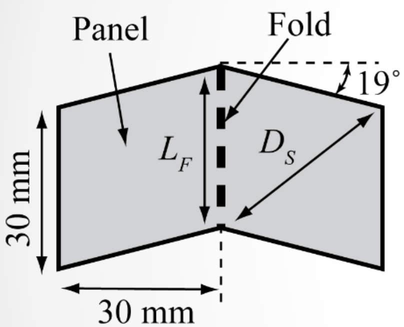
Nagasawa et al. (2003)

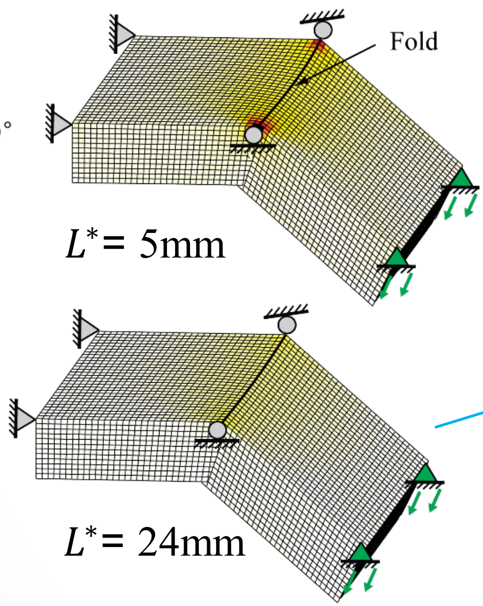


F Global Stiffness of Prescribed Folds

Maximum fold stiffness

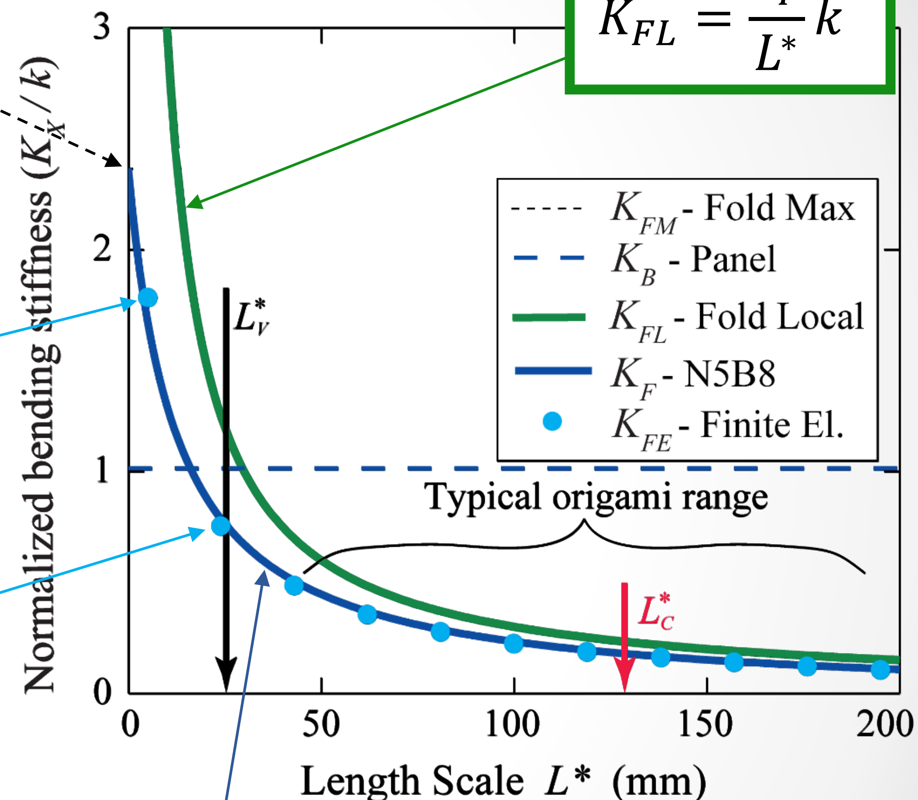
$$K_{FM} = K_B = \left(0.55 - 0.42\frac{0}{\pi}\right) k \left(\frac{D_S}{t}\right)^{1/3}$$





Local fold stiffness

$$K_{FL} = \frac{L_F}{L^*} k$$



Maximum and local stiffness in series

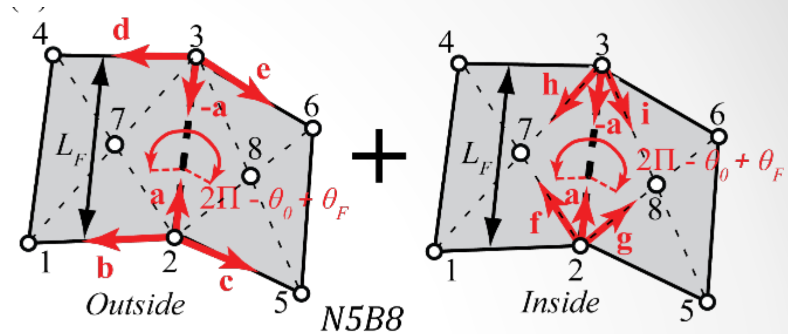
$$K_F = 1/\left(\frac{1}{K_{FL}} + \frac{1}{K_{FM}}\right)$$

Global fold stiffness for N5B8 model

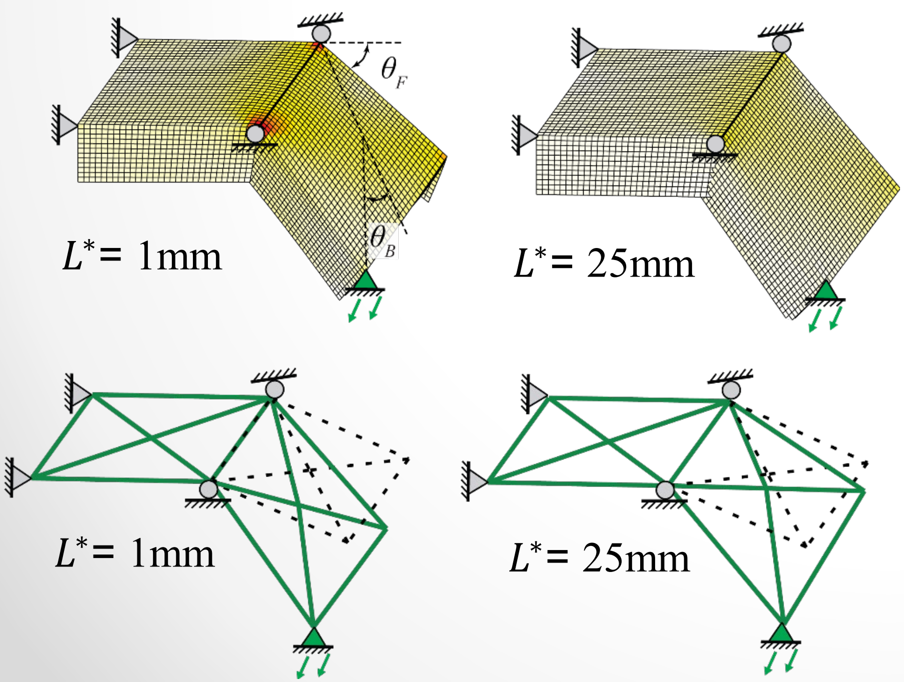
$$K_F = 1/\left(\frac{L^*}{L_F k} + \frac{1}{0.55k}\left(\frac{D_S}{t}\right)^{-1/3}\right)$$

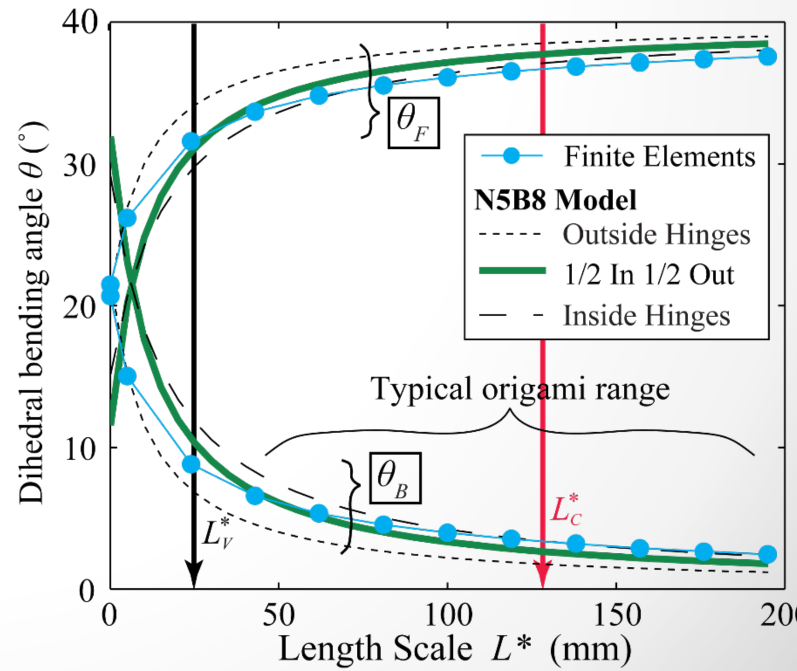
F Modeling Prescribed Fold Lines

- Fold line stiffness can be distributed on the outside and/or inside nodes of the N5B8 model

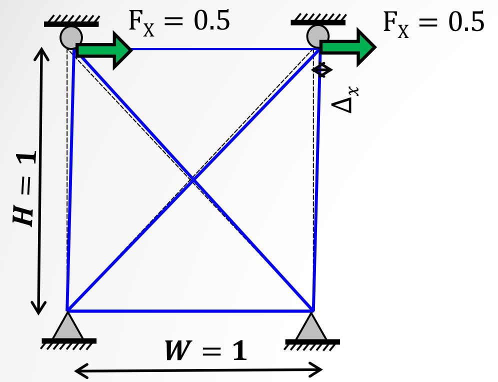


Asymmetric bending of folds (θ_F - top) vs. panels (θ_B - bottom)



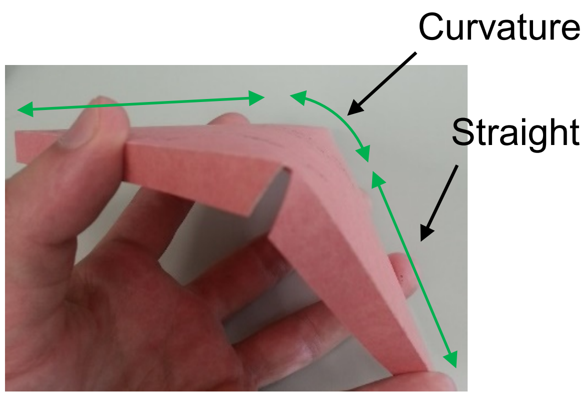


S Panel Shear & Stretching



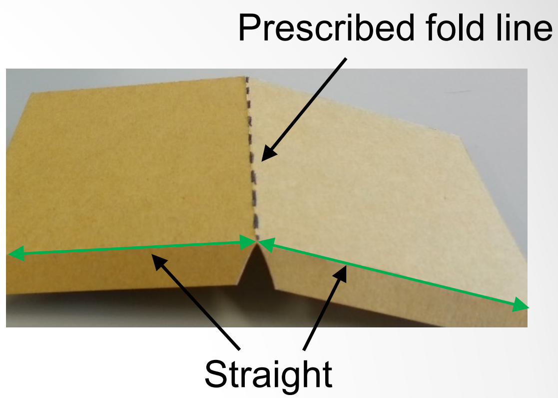
$$K_S = \frac{EA_{Bars}}{L}$$

B Panel Bending



$$K_B = \left(0.55 - 0.42\,\frac{\Sigma\alpha}{\pi}\right) k \left(\frac{L_2}{t}\right)^{\frac{1}{3}}$$

F Fold Bending



$$K_F = 1/\left(\frac{L^*}{L_F k} + \frac{1}{0.55k}\left(\frac{D_S}{t}\right)^{-1/3}\right)$$

- Scalability
- Model isotropy
- Geometric influence
- Material properties t, E, and ν

$$\mathbf{K} = \begin{bmatrix} \mathbf{C} \\ \mathbf{J}_B \\ \mathbf{J}_F \end{bmatrix}^T \begin{bmatrix} \mathbf{K}_S & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_F \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \mathbf{J}_B \\ \mathbf{J}_F \end{bmatrix}$$

Filipov E, Tachi T, and Paulino GH (2016) *Origami 6*

Filipov E, Liu K, Tachi T, and Paulino GH (2016) In Preparation

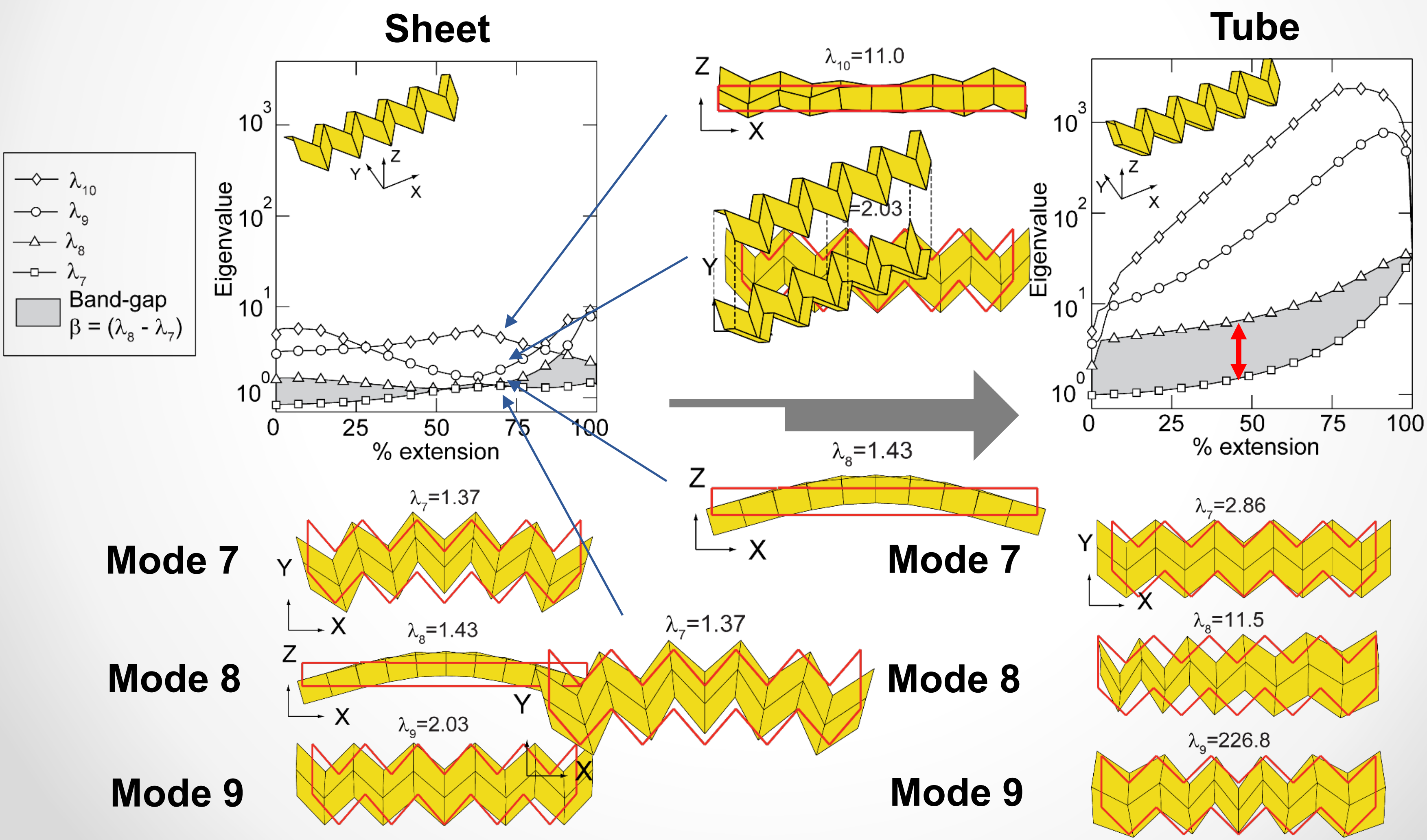
Eigenvalue Analyses

K = Stiffness matrix

M = Mass matrix

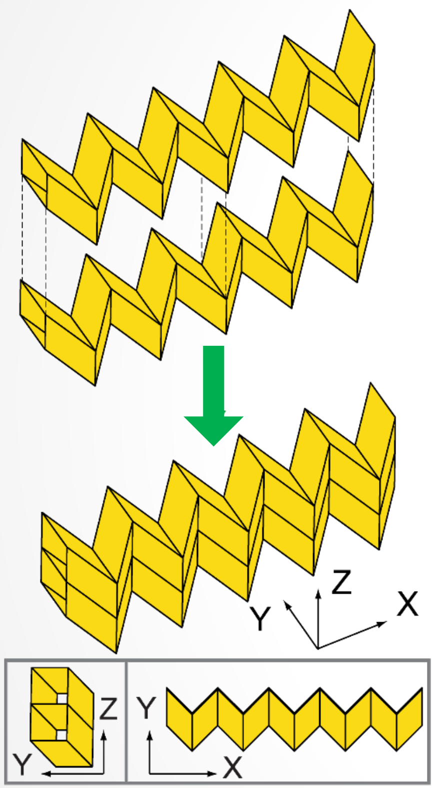
$$\mathbf{K}\mathbf{v}_i = \lambda_i \mathbf{M}\mathbf{v}_i \qquad i = 1, \dots, N_{dof}$$

Eigenvalue (λ_i) Eigen-mode ($\mathbf{v}_i$)

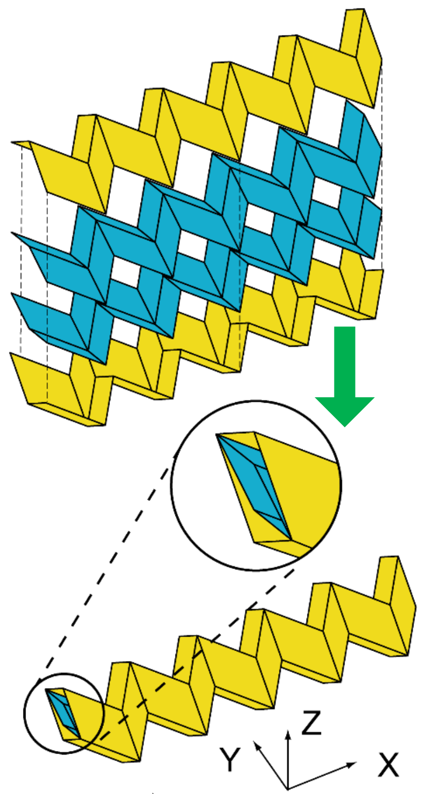


Tube Assemblages

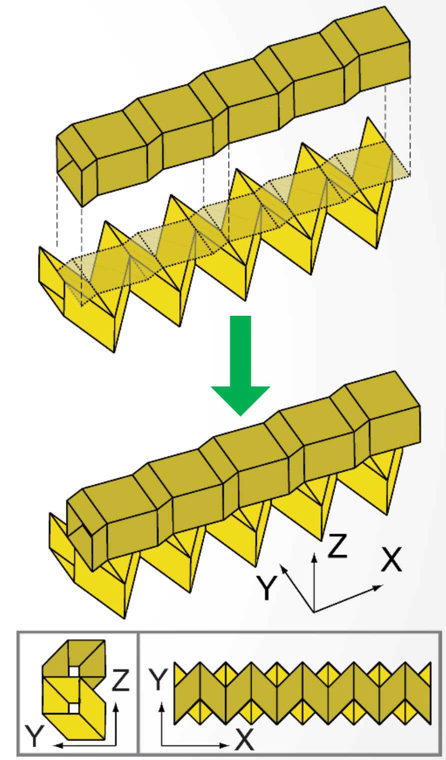
Aligned coupling

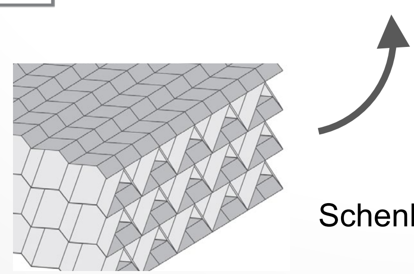


Internal coupling

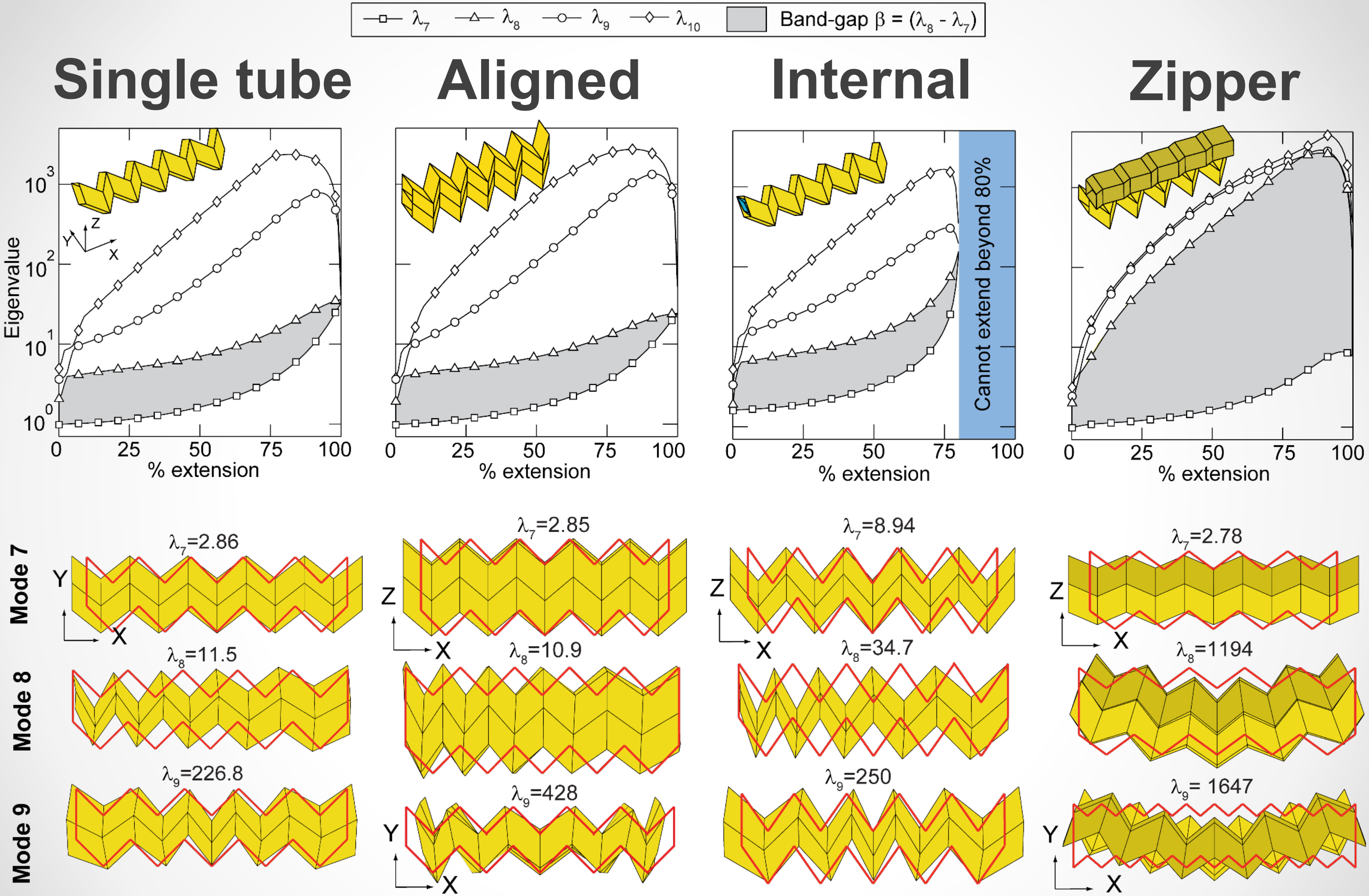


Zipper coupling





Schenk and Guest (2013)



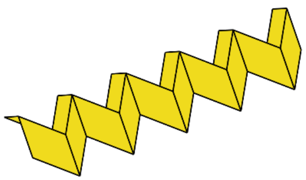
Filipov EF, Tachi T, and Paulino GH (2015) *PNAS* Vol. 112, No. 40

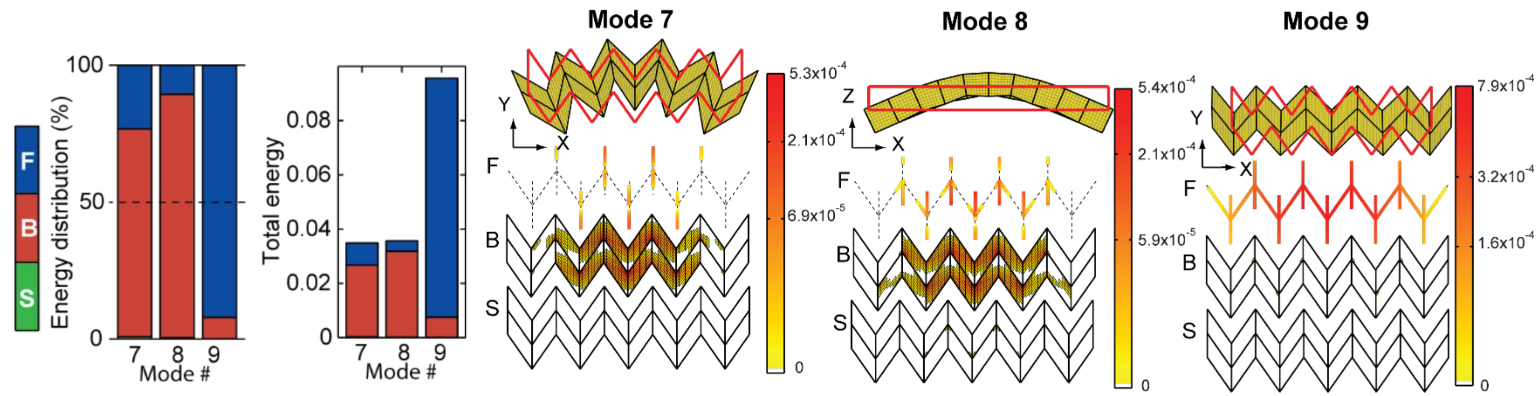
2015 Cozzarelli Prize

NATIONAL ACADEMY OF SCIENCES

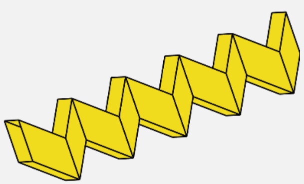
Energy Distribution

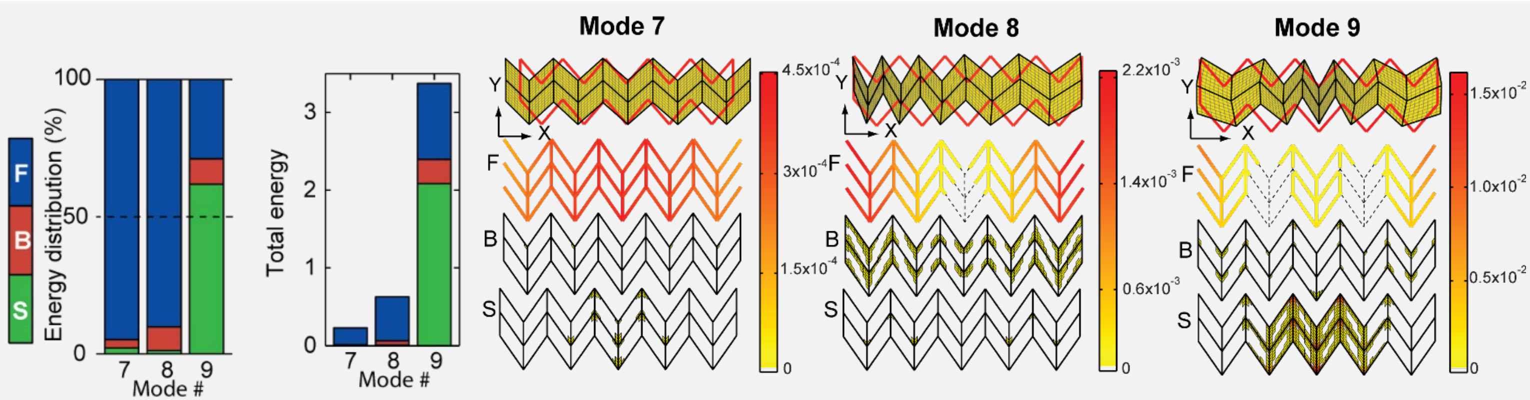
Sheet



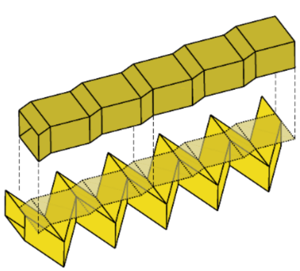


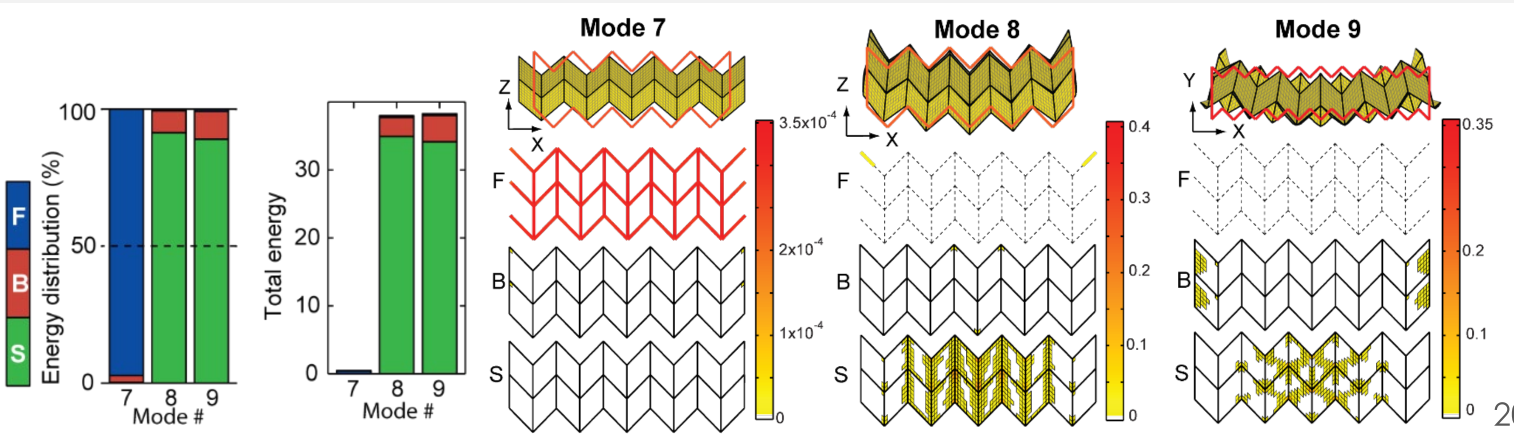
Tube





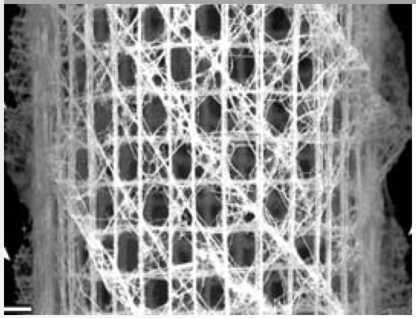
Zipper

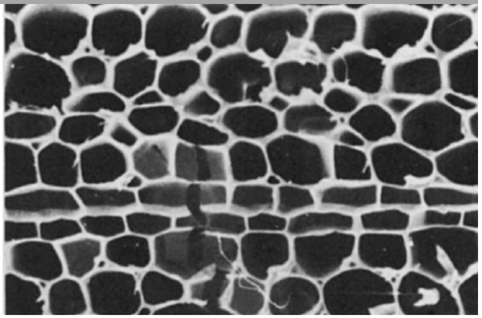




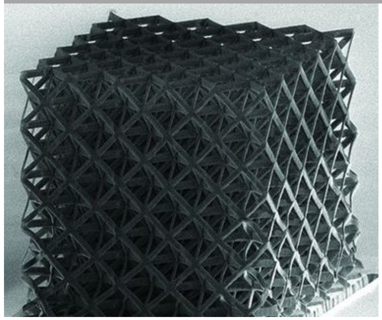
Cellular Assemblages as Metamaterials

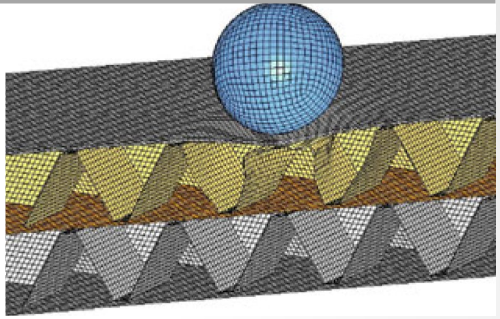
Nature


Aizenberg et al. (2005)


Ashby et al. (1985)

Engineering


Meza et al. (2014)

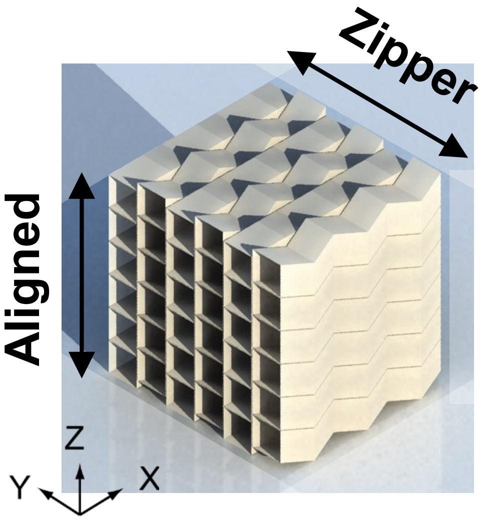

Heimbs (2013)

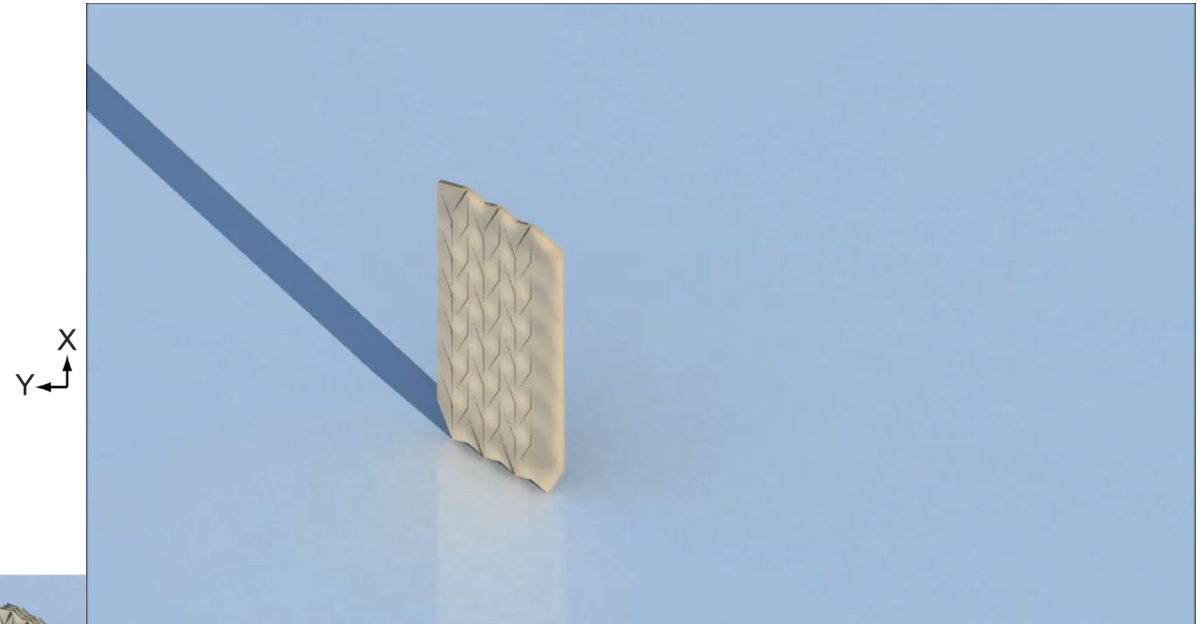
- Hierarchical properties (e.g. lattice systems)
- High stiffness to weight ratios
- Novel properties (auxetics or asymmetry)

With origami:

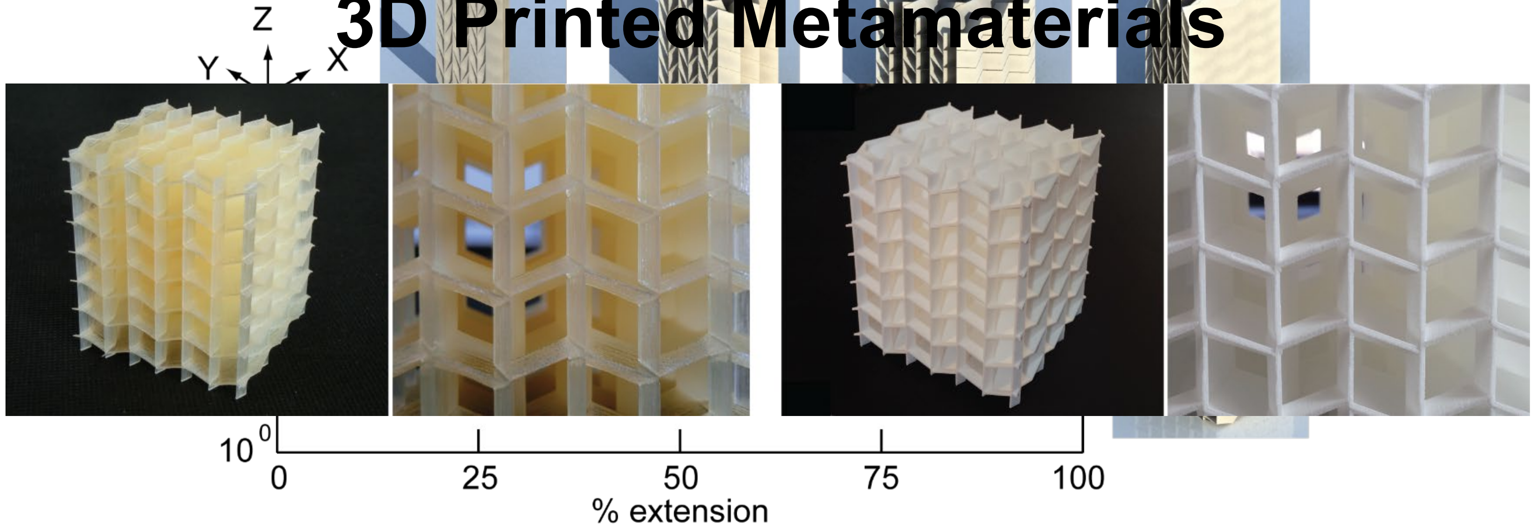
- Self-assembly
- Deployable
- Tunable characteristics

Zipper + Aligned Assemblage

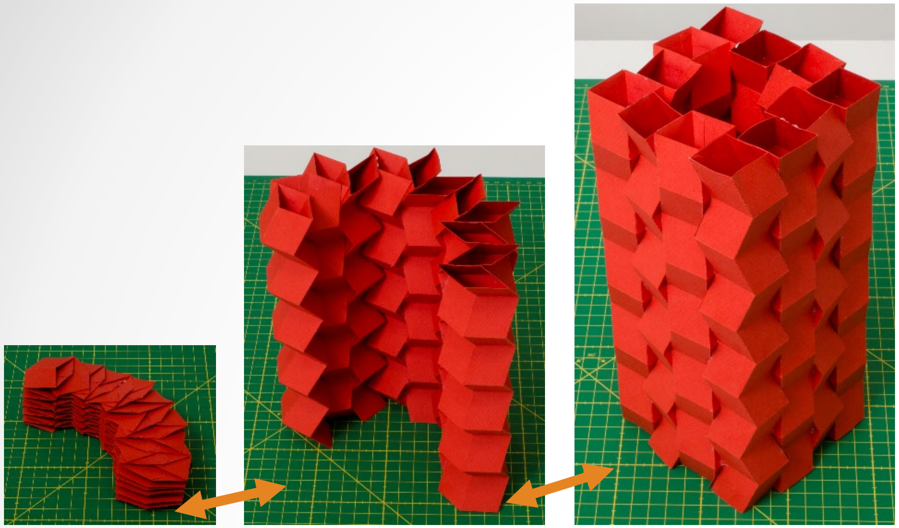




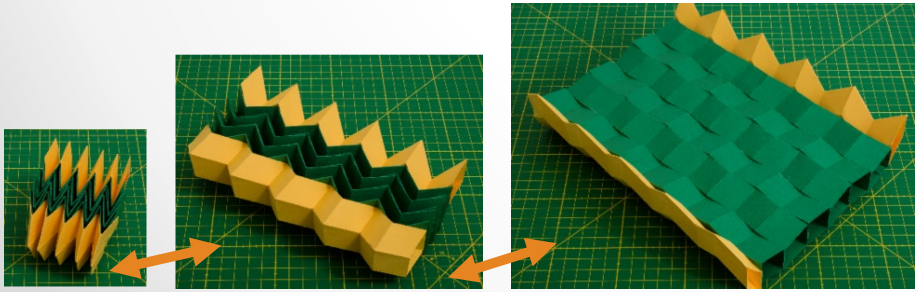
3D Printed Metamaterials



Self-Interlocking Structure

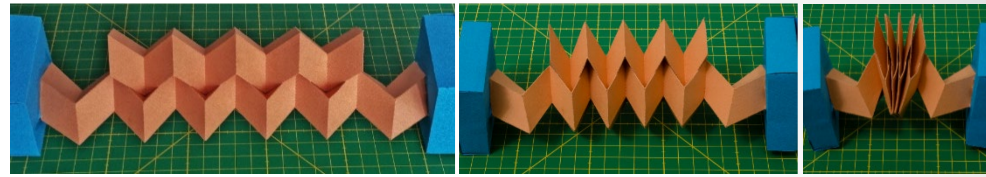


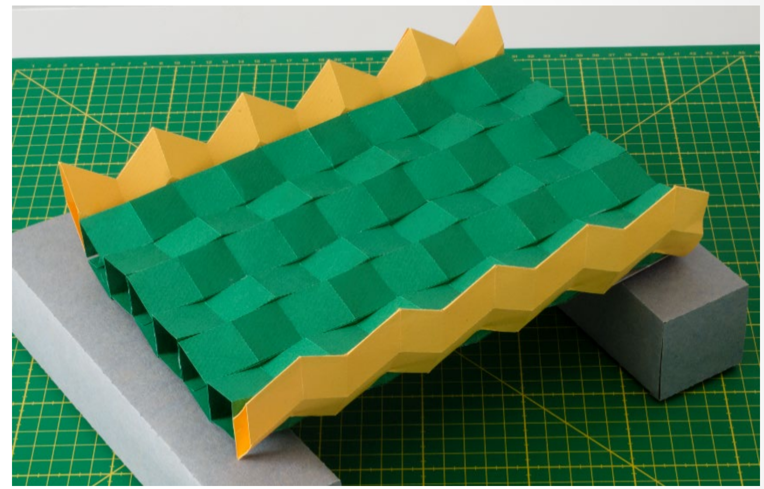
Bridge Structure

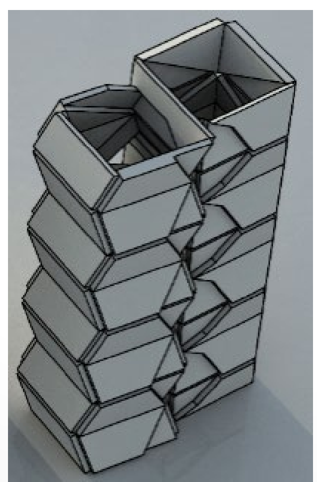


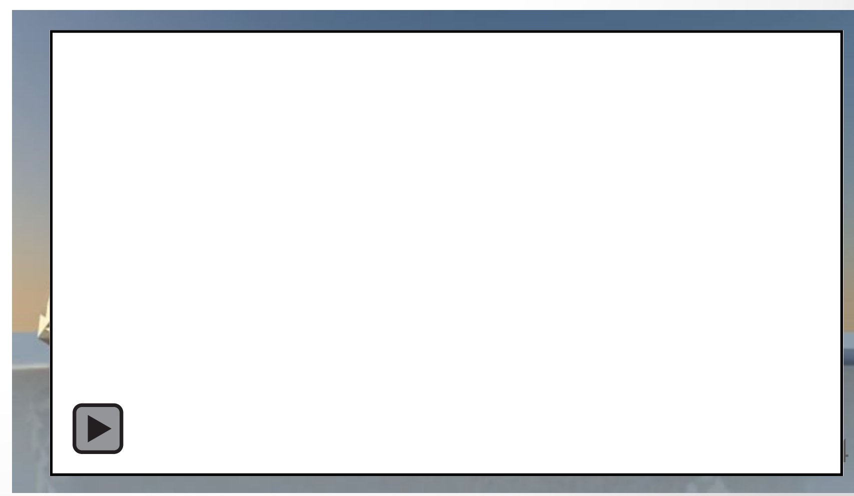
Extensions and Future of Zipper Tubes

- **Localized adaptations**
- **Geometric variations**
- **Tailored applications at different scales**
  - Thickness
  - Material
  - Fabrication
  - ….









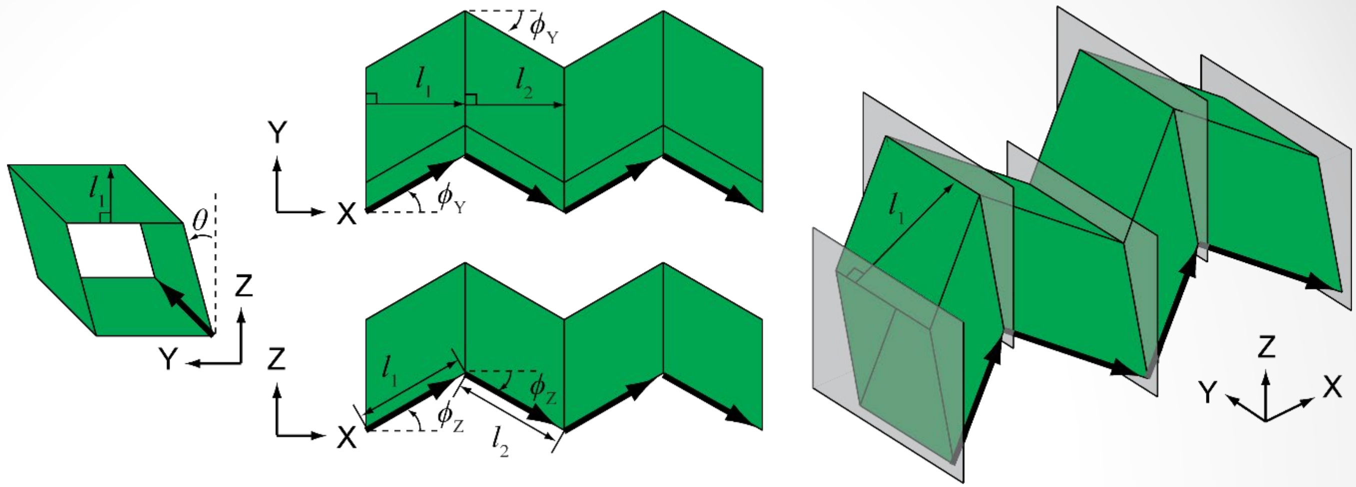
Generalized Definition for Origami Tube

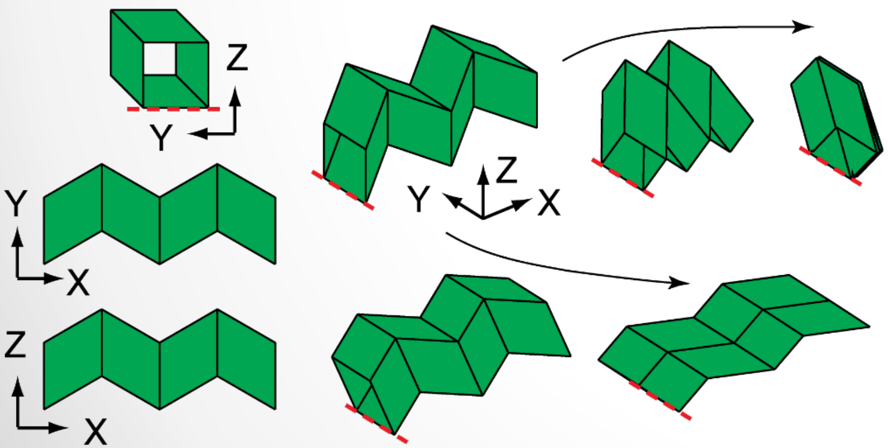
Projection

ϕ_Z and ϕ_Y

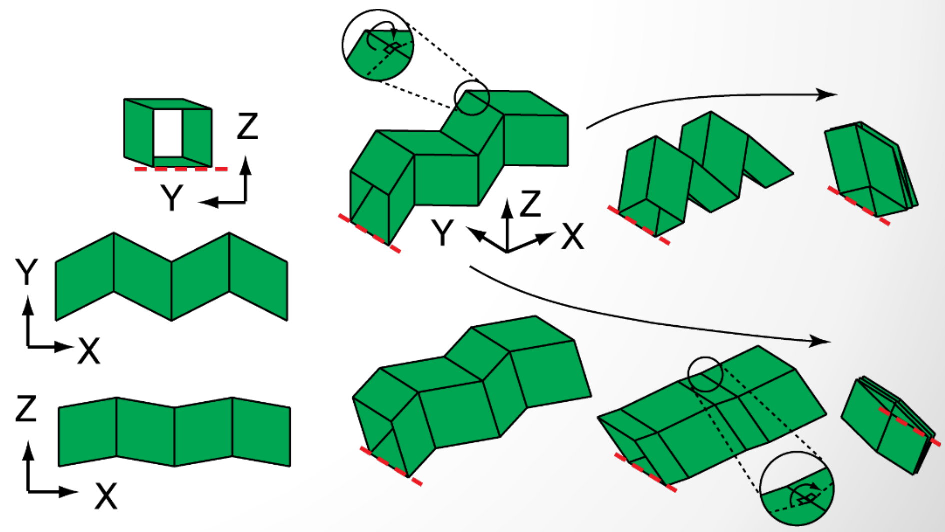
Cross-section

θ



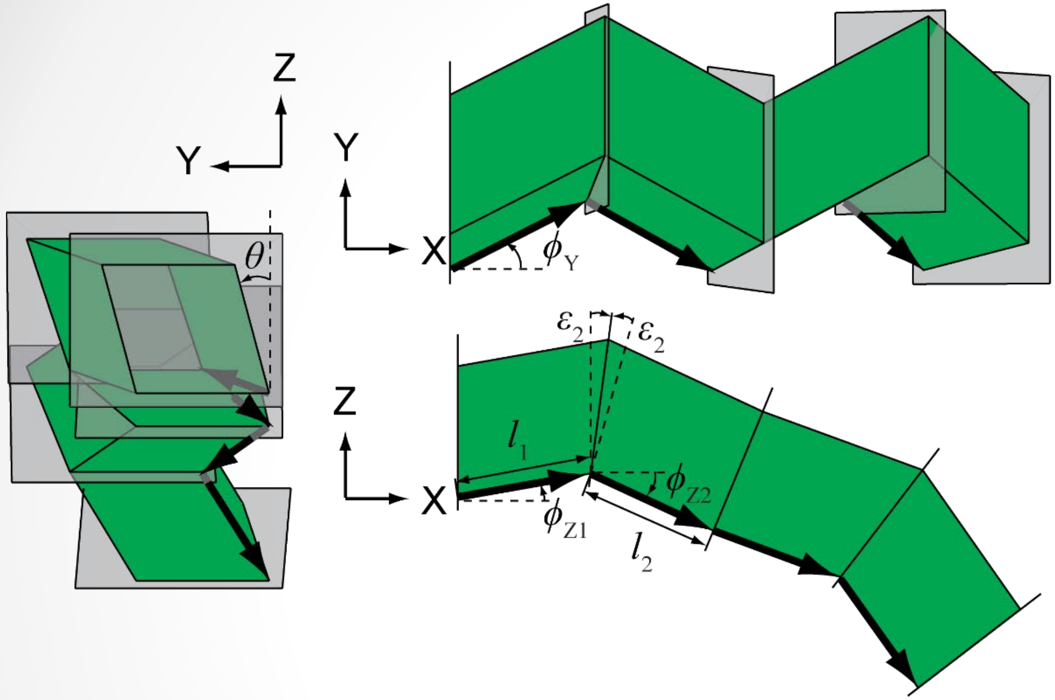


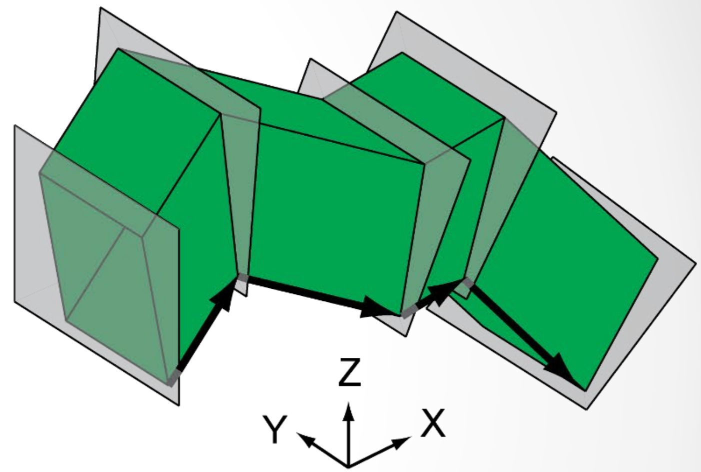
$\phi_Z = \phi_Y = 30^\circ$



$\phi_Z = 10^\circ < \phi_Y = 30^\circ$

Non-straight Origami Tube





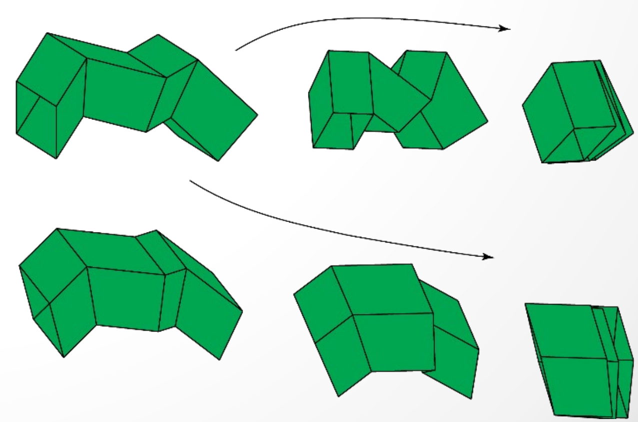
Projection

$$\phi_{Z1} \quad | \quad \phi_Y \quad | \quad \varepsilon_1,\ \varepsilon_2 \ldots \varepsilon_i$$

$$\phi_{Z2} = \phi_{Z1} + 2\varepsilon_2 \quad \ldots \text{ etc.}$$

Cross-section

θ



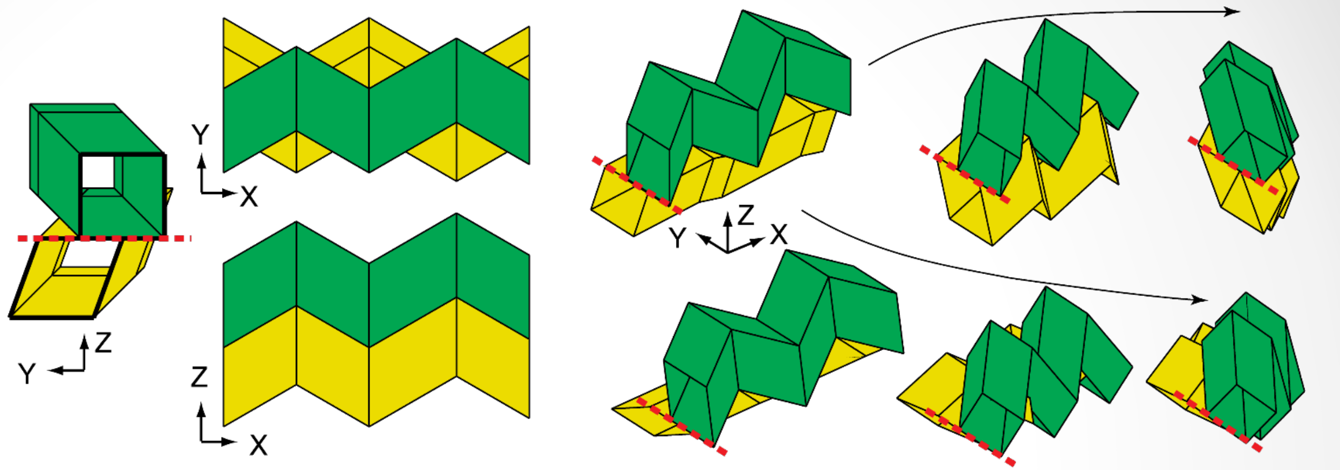
Generalized Definition for Coupled Tubes

Projection

ϕ_Z ϕ_{YT} ϕ_{YB}

Cross-section

θ_T θ_B

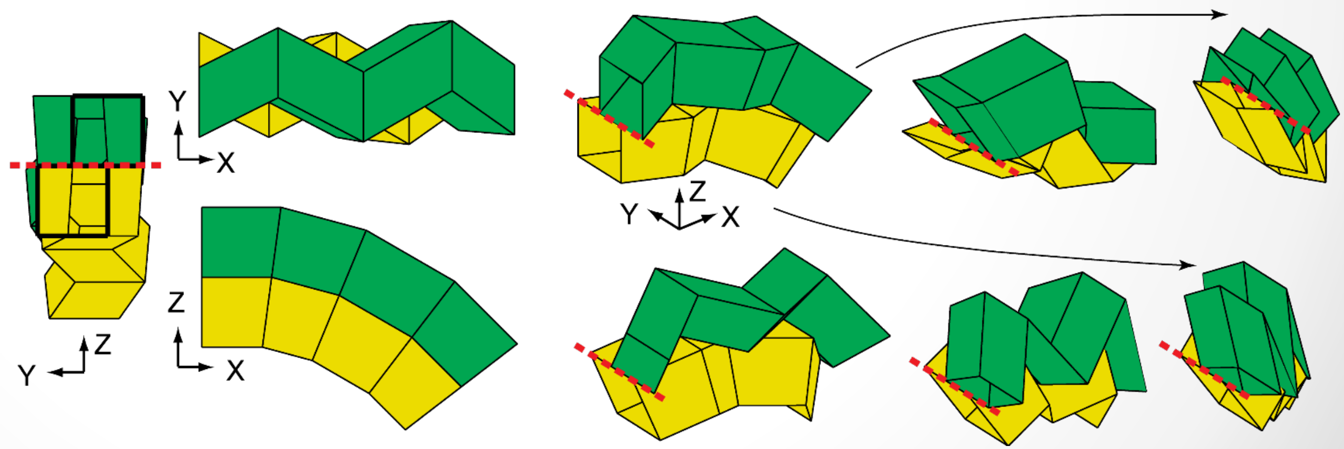


Projection

ϕ_Z $\phi_{YT} = -\phi_{YB}$

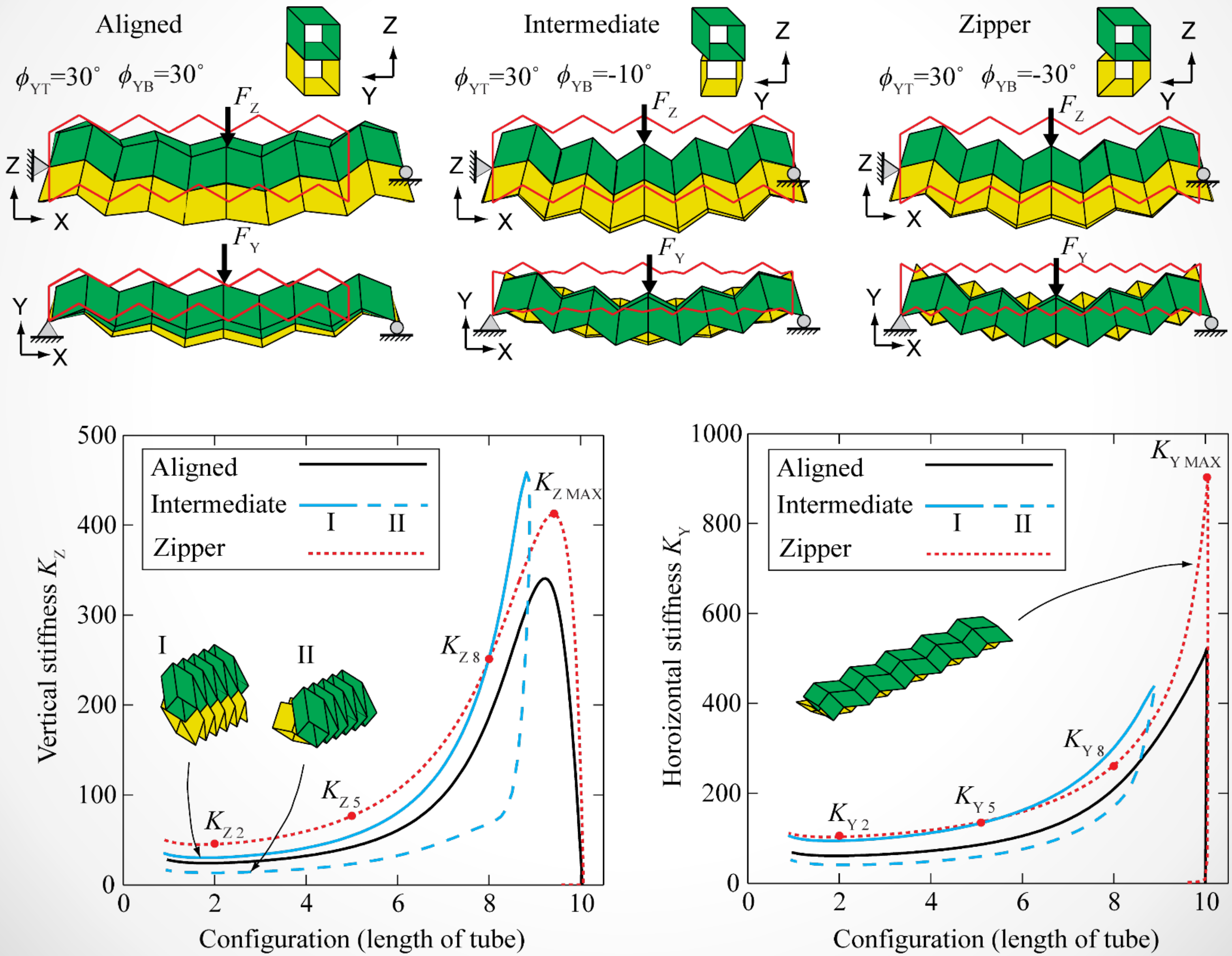
Cross-section

$\theta_T = -\theta_B$

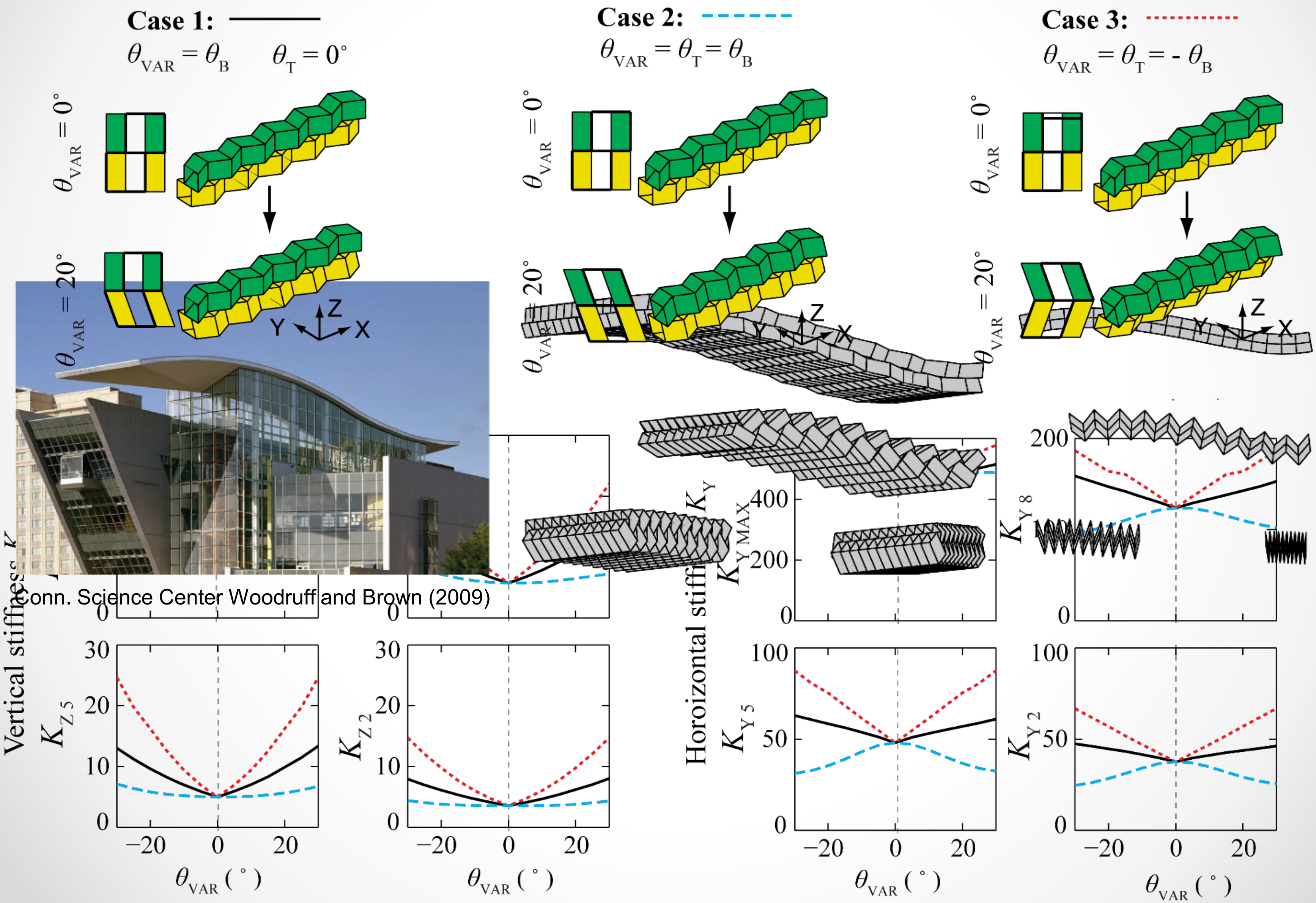


Tachi T., Filipov, E.T., and Paulino, G.H. (2015) *Proc. IASS*

Stiffness of Straight Coupled Tubes

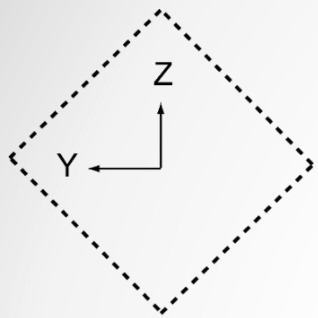


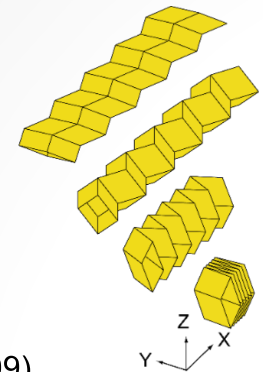
Slabs and Arches with Flat Surface



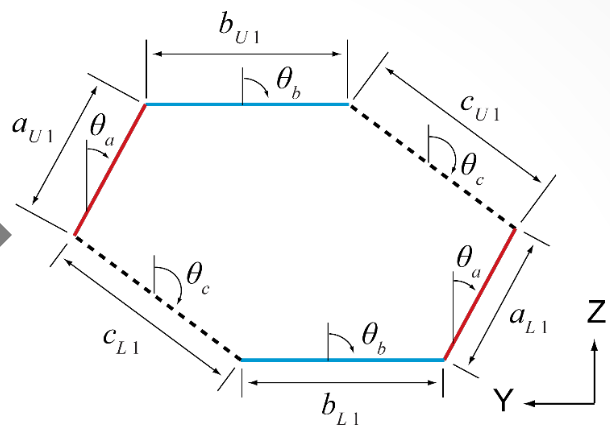
Conn. Science Center Woodruff and Brown (2009)

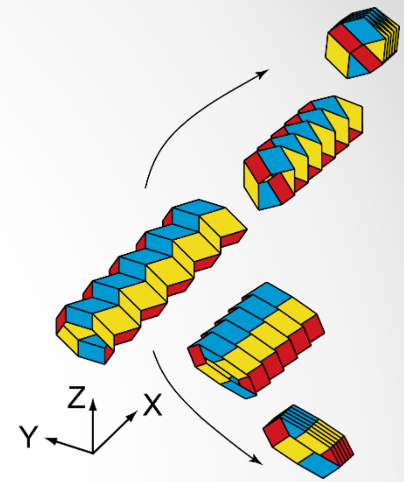
Extend to Polygonal Cross-Sections



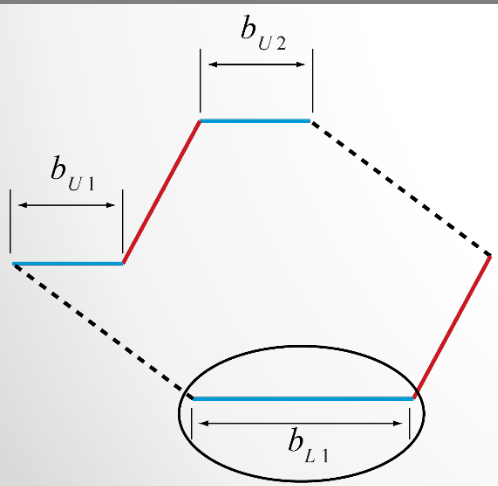


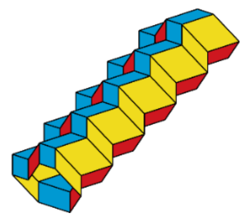
Tachi and Miura (2009)

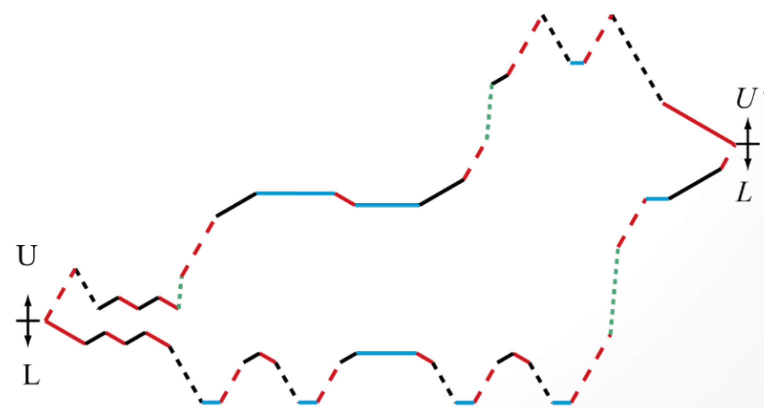


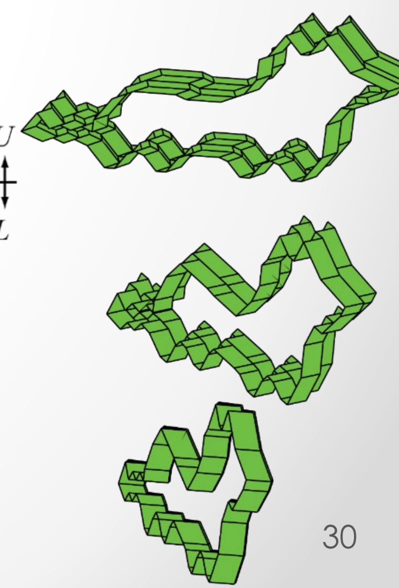


Partition and Re-arrange





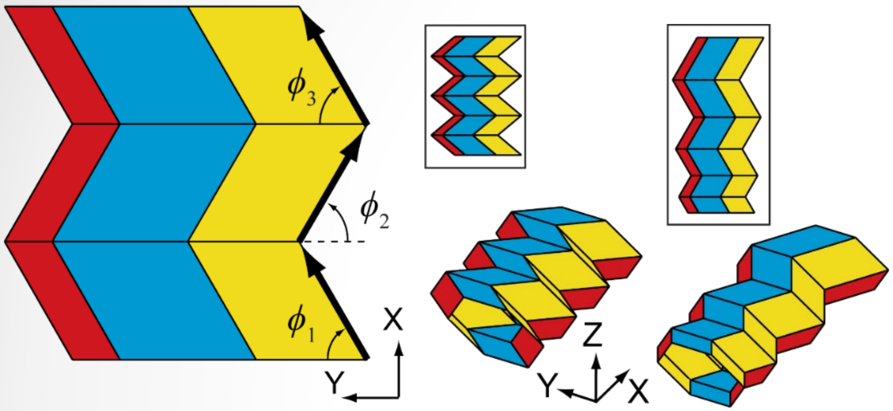




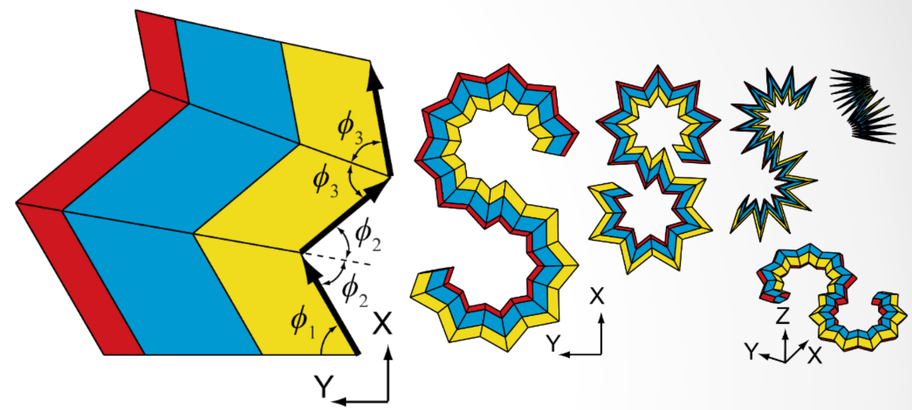
Filipov, Tachi, and Paulino (2016)
Proceeding of the Royal Society – A, Vol. 472, No. 2185

Projection Definitions

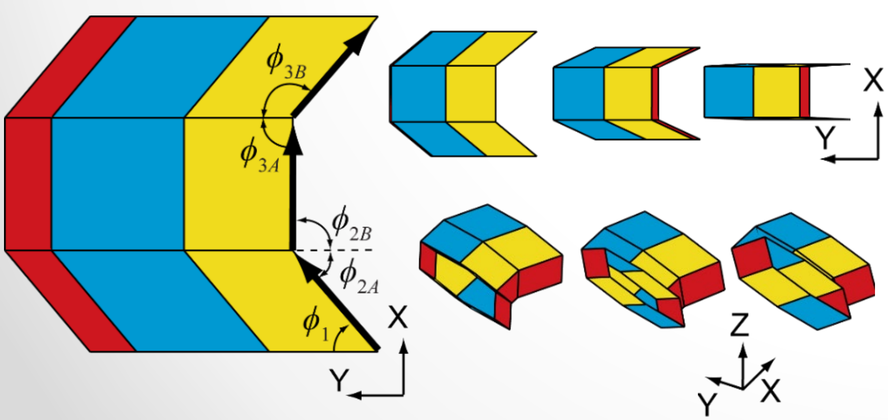
Constant X-Y



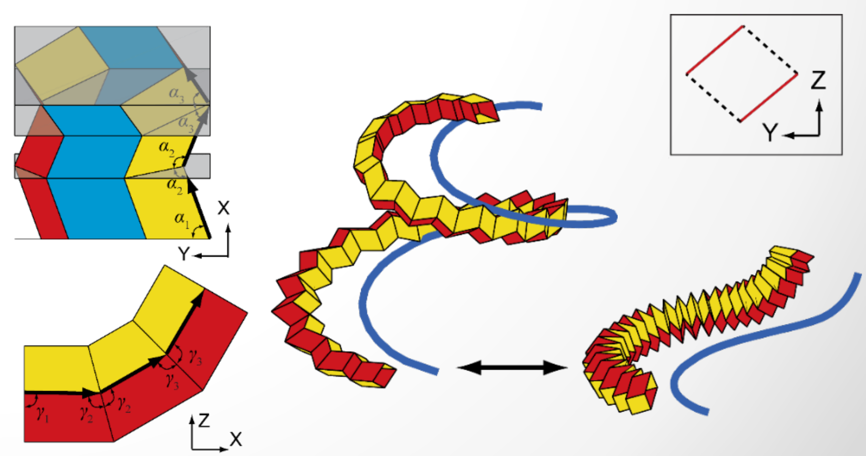
Symmetric X-Y



Non-symmetric X-Y



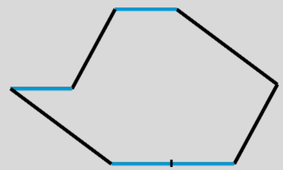
Symmetric X-Y-Z

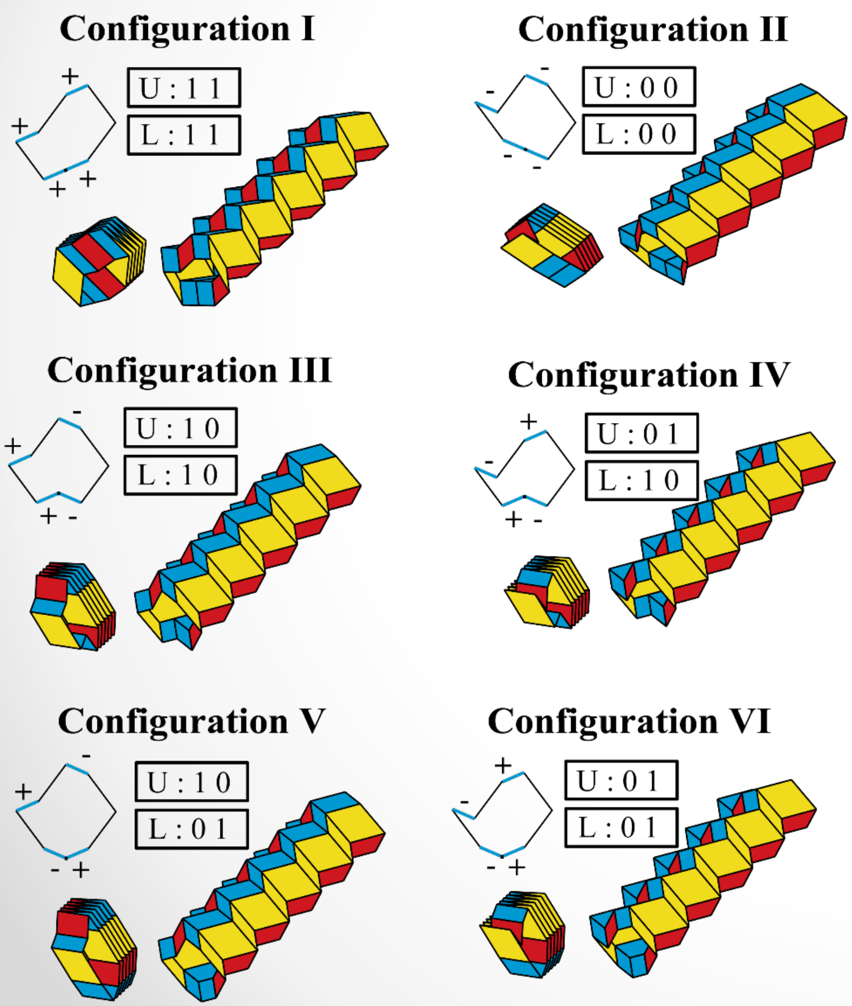


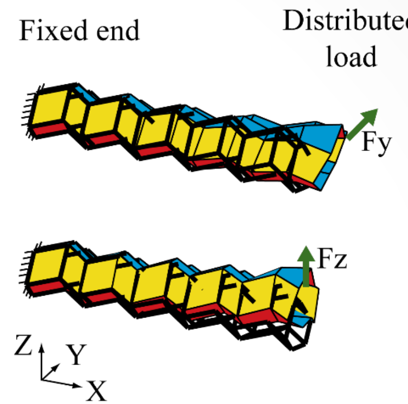
Filipov et al. (2016) *PRSA*

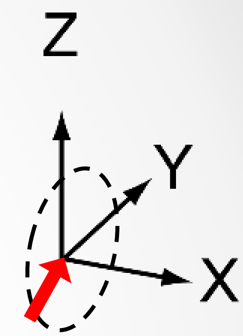
Reconfiguration with n = 2 Switches

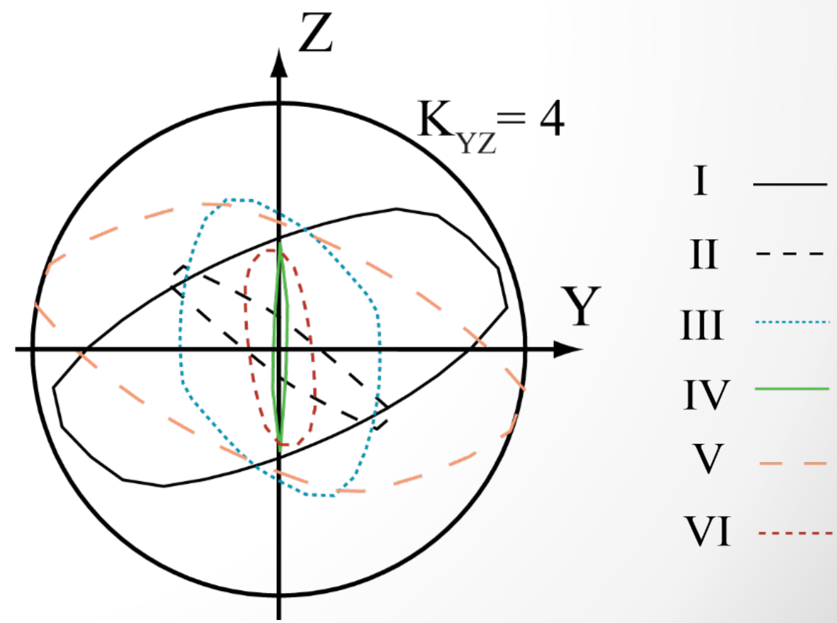
n = 2 switches











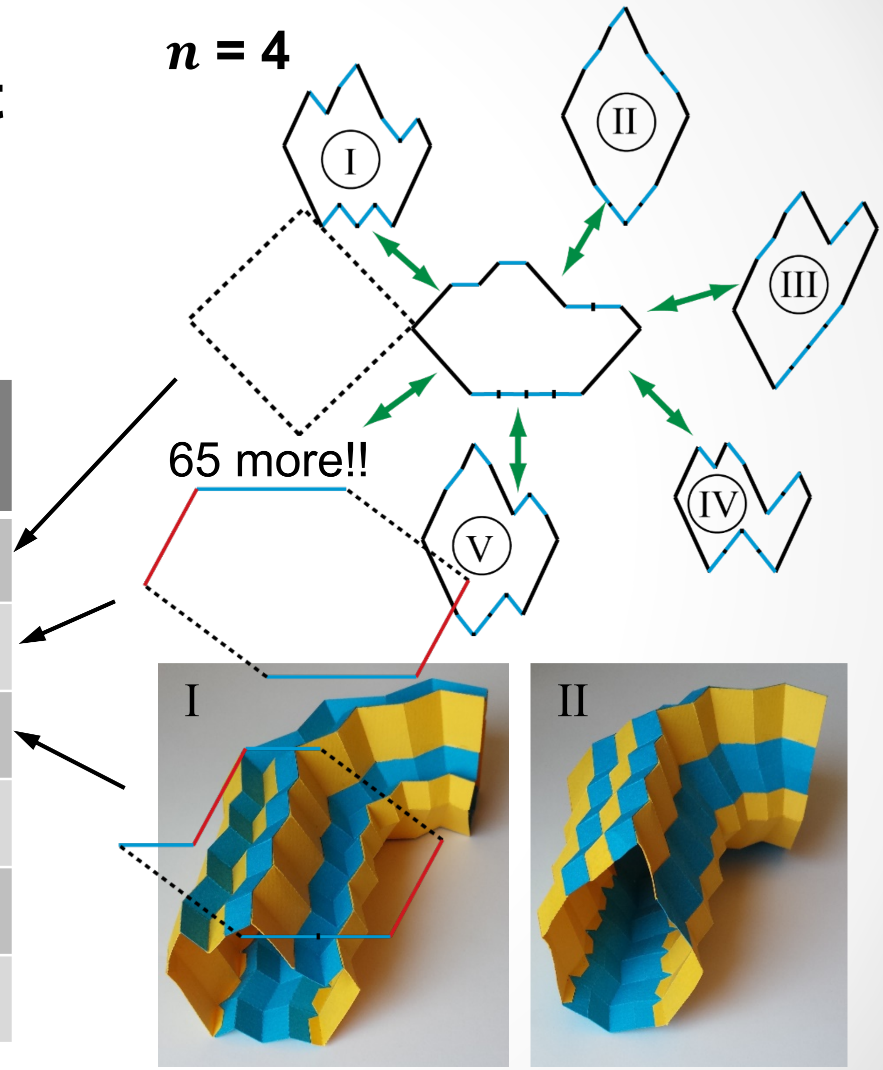
Filipov et al. (2016) *PRSA*

Reconfiguration with More Switches

Central Binomial Coefficient

$$\# \text{ Configurations} = \frac{(2n)!}{(n!)^2}$$

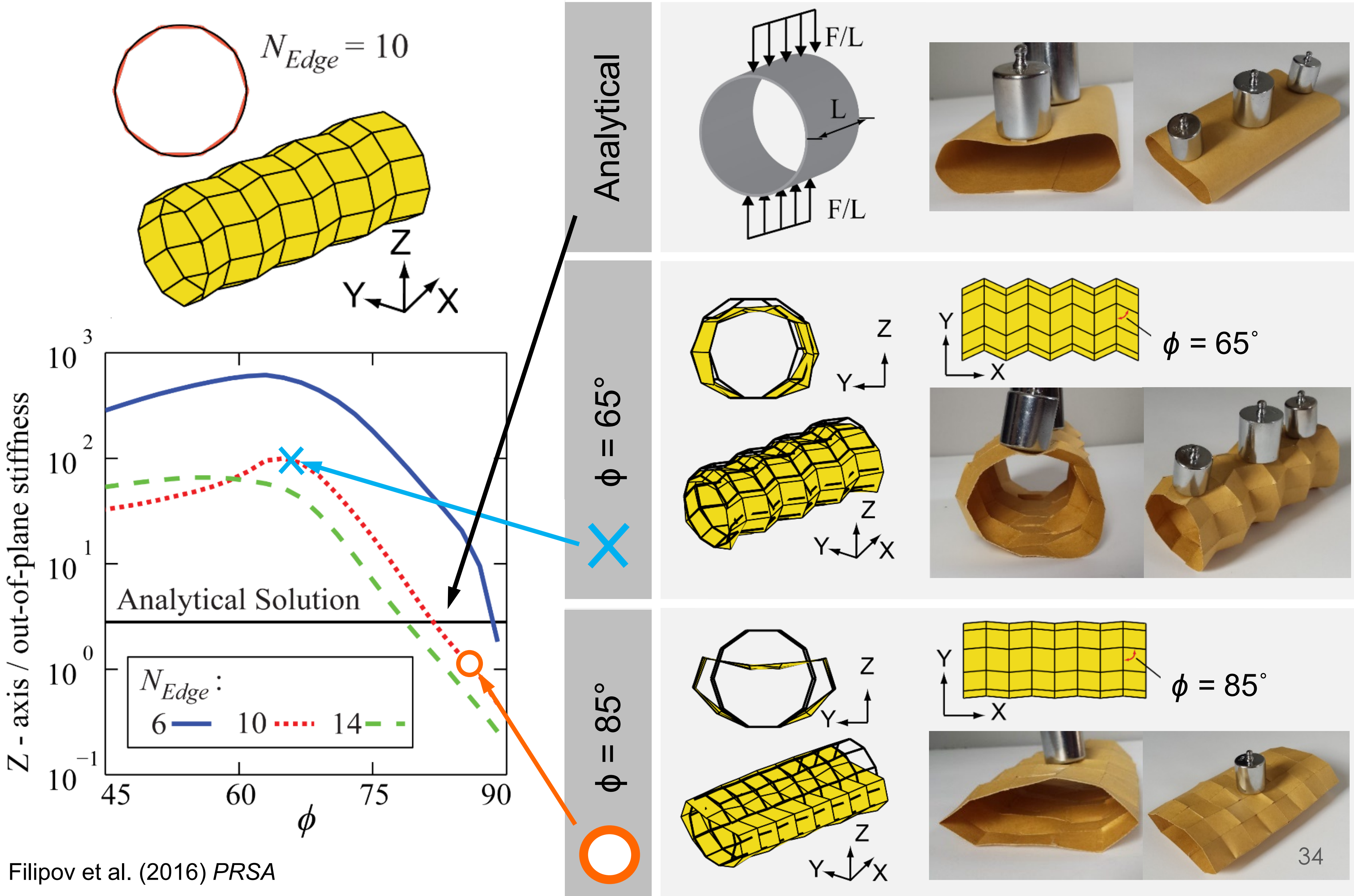
Switches	# Configurations
n = 0	1
n = 1	2
n = 2	6
n = 3	20
n = 4	70
n = 5	252



Physical model, n = 4 switches

Filipov et al. (2016) *PRSA*

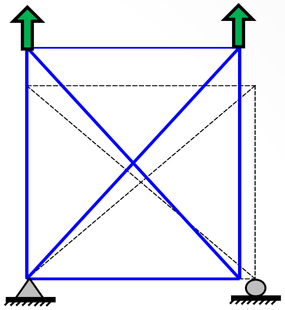
Out-of-Plane Compression of a Pipe

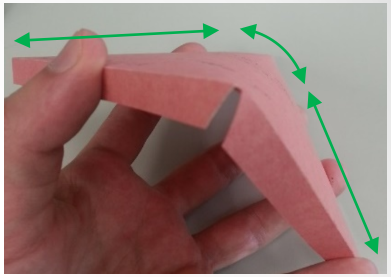


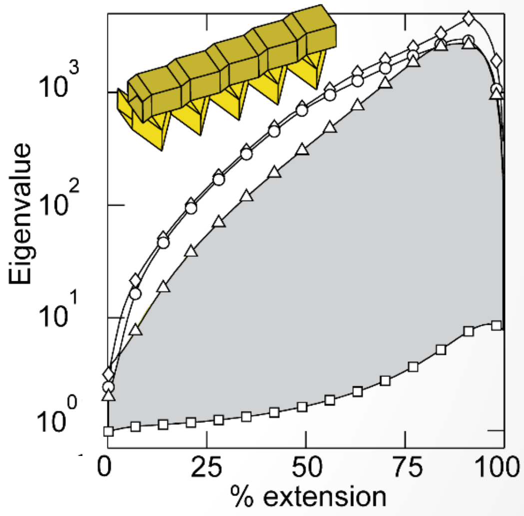
Filipov et al. (2016) *PRSA*

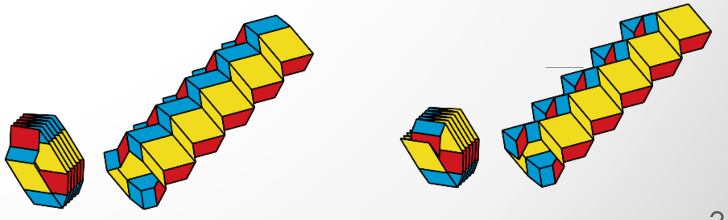
Summary of Ph.D. Research

- **Improved structural analysis for origami**
- **Influence of geometry on origami stiffness *locally* and *globally***
- **Zipper-coupled systems *engage thin sheet in shear/stretching***
- **Geometric variation of tube cross-sections and profiles**
- **Structural tuning through reconfiguration**









Thank you!

Acknowledgments:

filipov1@illinois.edu

www.efilipov.com