US-South America Workshop
Mechanics and Advanced Materials
Research and Education

An approach to limit states in advanced materials

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16 DSc
4 MSc
8 Scientific Initiation Program

- Structural Integrity
  - Plasticity and Viscoplasticity
  - Fatigue
  - Damage
  - Limit States
- Structural Dynamic
- Advanced Materials

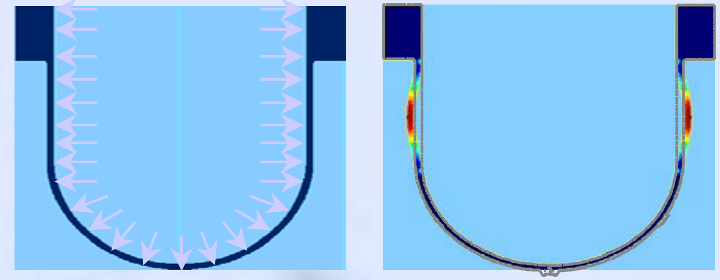
Outline

- **Limit States:**
  - – **Limit analysis : Model and adaptive remesh**
  - – **Shakedown for FGM beam**

Limit Load

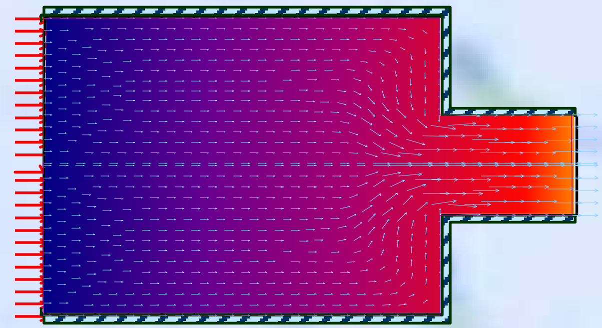
Plastic Collapse

Structural Analysis



Plastic Flow

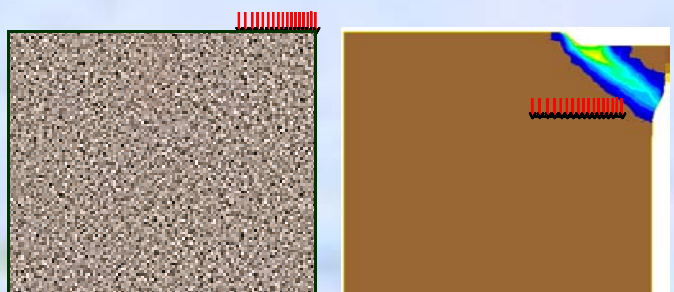
Metal Forming

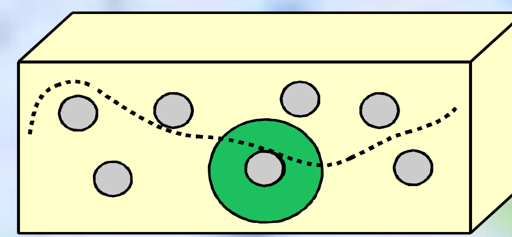


Failure Criterion

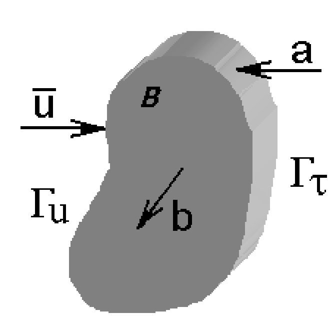
Geomechanics; - Bearing capacity

Composites- Failure Prediction.





Limit Analysis of a continuum



Configuration in t

$\Gamma = \Gamma_u \cup \Gamma_\tau$

$$D^p = \mathcal{D}v \qquad v \in V$$

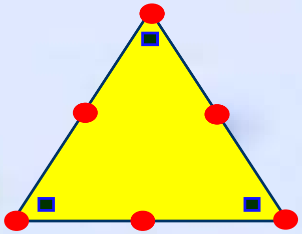
$$T \in S(\alpha F)$$

$$T \in \partial X(D^p)$$

$$\alpha = \inf_{v \in V} \sup_{T \in W'} < T, \mathcal{D}v > \quad \Bigg| \quad \begin{array}{c} < F, v > = 1 \\ T \in P \end{array}$$

$$P = \{ T \in W' \mid \quad f(T) \leq 0 \quad in\ \mathcal{B} \}$$

Limit Analysis: Discrete Model - Mixed Elements



- Nodes for velocities interpolation (QUADRATIC)
- Nodes for stress interpolation (linear)

$$\alpha = \min_{\mathbf{v}\in\Re^{\mathbf{n}}} \max_{\mathbf{T}\in\Re^{\mathbf{q}}} \mathbf{T}\cdot B\mathbf{v} \;\Bigg|\; \begin{array}{l} F\cdot\mathbf{v} = \mathbf{1} \\ f(\mathbf{T}) \leq \mathbf{0} \end{array}$$

$$B\mathbf{v} - \nabla f(\mathbf{T})\dot{\lambda} = 0$$

$$B^T\mathbf{T} - \alpha F = 1$$

$$F\cdot\mathbf{v} = \mathbf{1}$$

$$f_j(\mathbf{T})\dot{\lambda}_j = 0 \qquad f_j(\mathbf{T}) \leq 0 \qquad \dot{\lambda}_j \geq 0$$

$$j = 1,\ldots,m$$

Algorithm

Newton-like formula associated with the set of all equalities included in the optimality conditions, followed by a step relaxation and stress scaling in order to preserve the plastic admissibility constraint.

Estimators based on derivatives recovery

The interpolation error as an indicator of the approximate solution

$$u(x) = \overbrace{u(x_o) + \nabla u(x_0) \cdot (x - x_o)}^{\simeq u_h} + \frac{1}{2}\,\mathbf{H}(u(x_0))\,(x - x_0)(x - x_0) + \ldots$$

$$\|u - u_h\|_{L^p(\Omega)} \simeq C \|\mathcal{H}_R(u_h(x))(x - x_0) \cdot (x - x_0)\|_{L^p(\Omega)}$$

$H_r(u_h)$ is the recovered Hessian matrix

Error estimator

Local error

$$\eta_T = \left\{ \int_{\Omega_T} \left[G(u_h(x_0))\,(x - x_0) \cdot (x - x_0) \right]^p d\Omega \right\}^{\frac{1}{p}}$$

$$G = Q\,\Lambda\,Q^T$$

***H**_r* **- eigenvectors matrix**

error direction

absolute value of the *H_r* eigenvalues**

value of error

Global error

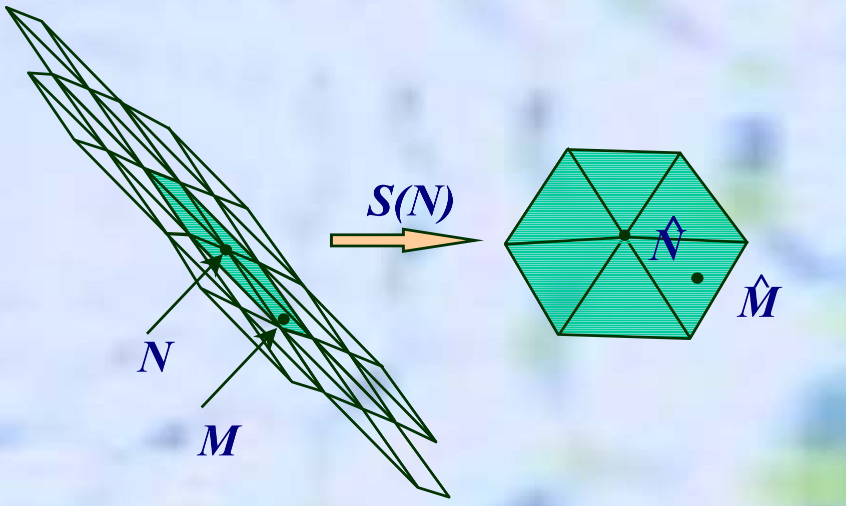
$$\eta = \left\{ \sum_{T \in \mathcal{T}_h} \eta_T^p \right\}^{\frac{1}{p}}$$

Derivative Recuperation: Weighted Average or Patch Recovering

In order to approximate representing strong variations in the derivatives the adapted mesh becomes oriented by means of stretching its elements in the direction of maximum curvature of function graph

$$\mathbf{S}(N) = \frac{1}{s(N)\cdot h(N)}\mathbf{e}_1 \otimes \mathbf{e}_1 + \frac{1}{h(N)}\mathbf{e}_2 \otimes \mathbf{e}_2$$

$$\mathbf{x}^*_{\mathbf{M}} = S(N)\,(x_M - x_N)$$



Stretched Mesh and the transformed domain defined by advancing frontal technique

First Derivative Recovery Algorithm

For each node N of the mesh:

1 . Define the patch associated to N.

2. For known $s(N)$, $h(N)$, e_1 and e_2, built the metric tensor S(*N*).

3. Transform the elements of the patch.

4. In each element compute the grad(u_h).

5. Compute recovered gradient grad$_r$(u_h).

6. Transform grad$_r$(u_h) to the original domain by

$$\nabla_R u_h(N) = S^T grad_R u_h(N)$$

Taking the first derivative as a new field we reapply the algorithm to recover the Second derivative

Adaptive Procedure

1.For each element compute the local error η_T and then the global error η

2.Given *Nel* in the new mesh, compute the expected local error indicator, equally distributed on all elements, by

$$\eta^* = \frac{\eta}{\sqrt{Nel}}$$

The decreasing or increasing rate of element size is estimated by

$$\beta_T = \left(\frac{\eta^*}{\eta_T}\right)^{\frac{1}{3}}$$

This parameter at nodal level $\beta(N)$ is computed by the same approach adopted for the recovering the derivatives.

4.Compute the size of the new element h_{k+1} and the stretching $s(N_k)$ node N, by

$$h_{k+1}(N) = \beta(N)\, h_k(N).$$

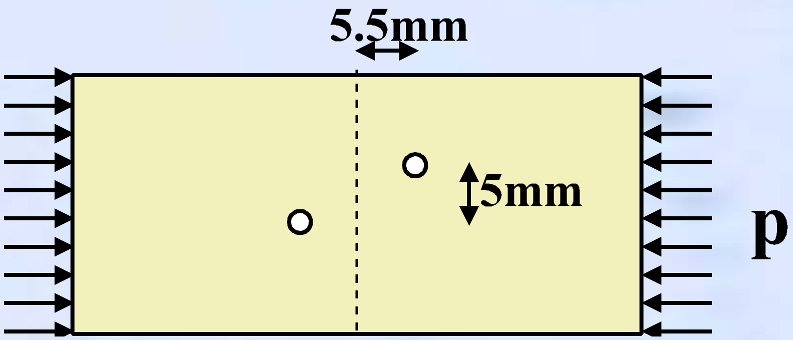
$$s(N_k) = \sqrt{\frac{|\lambda_2|}{|\lambda_1|}}$$

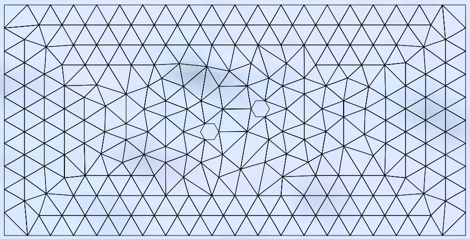
where λ_1 and λ_2 are the absolute eigenvalue of Hessian matrix.

5. h_{k+1} _*scaling*

$$h_{k+1} \leftarrow \sqrt{\frac{Nel_{new}}{Nel}}\, h_{k+1} \qquad \text{with} \qquad Nel_{new} = \frac{4}{\sqrt{3}} \int_\Omega \frac{2}{sh^2} d\Omega$$

Plate with imperfections (Diez, 1999)



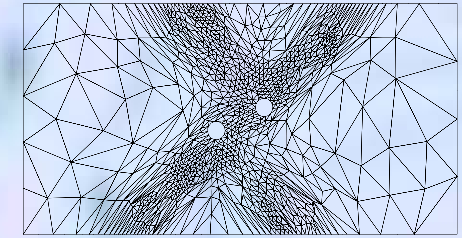


Initial mesh

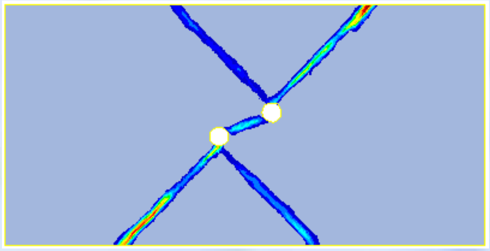
1908 dof

Plane Strain

$p_c = 1.0350\ \sigma_Y$

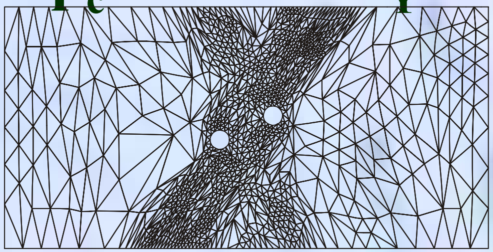


Final mesh:7064

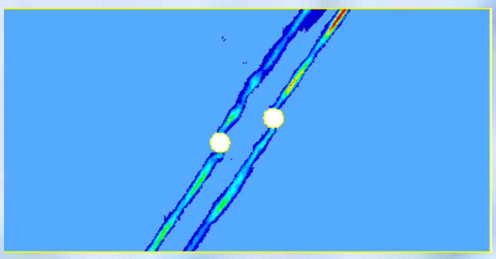


Plane Stress

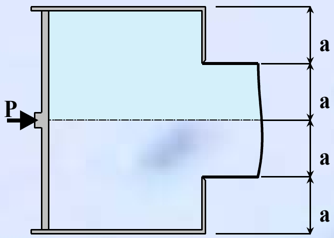
$p_c = 0.9123\ \sigma_Y$

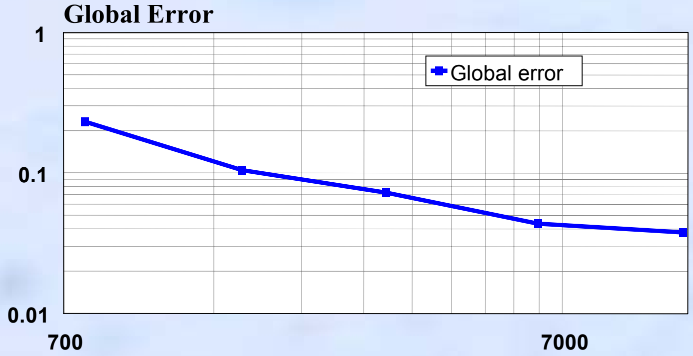


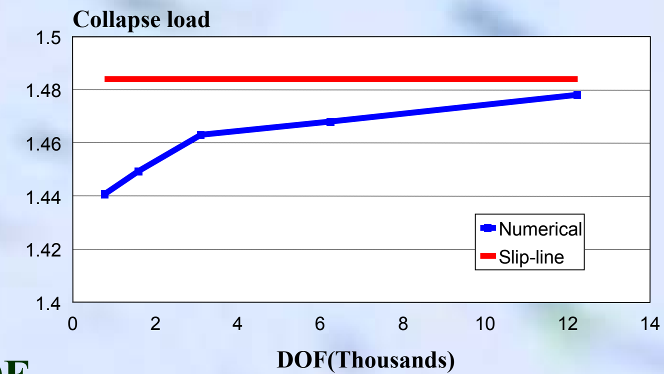
Final mesh: 8194 dof



Frictionless extrusion through a square die – Reduction 1/2

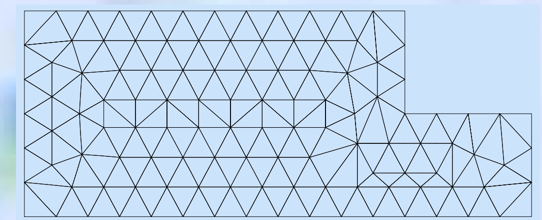


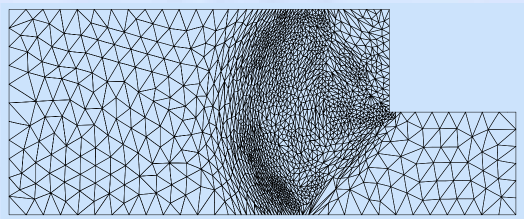


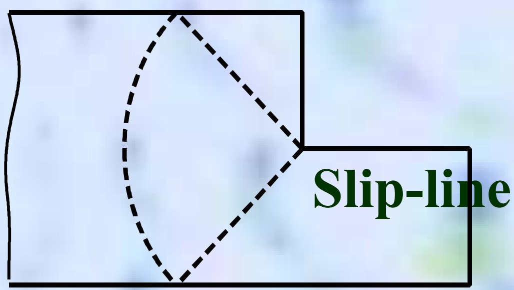


Initial - 773 DOF

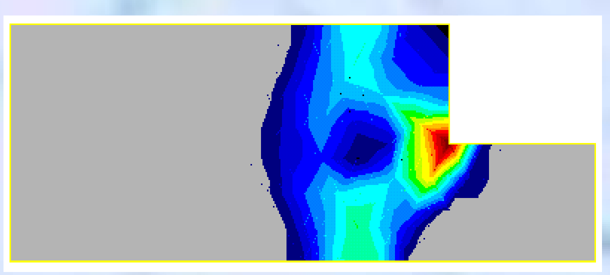
After five Steps - 12247 DOF



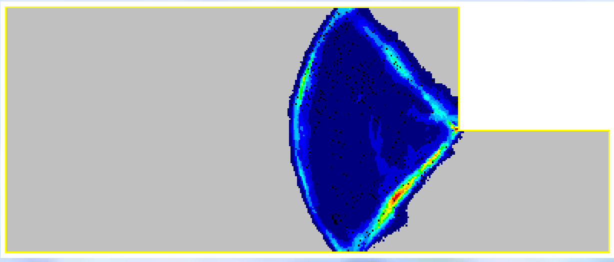




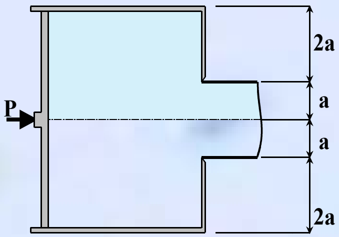
plastic multiplier field

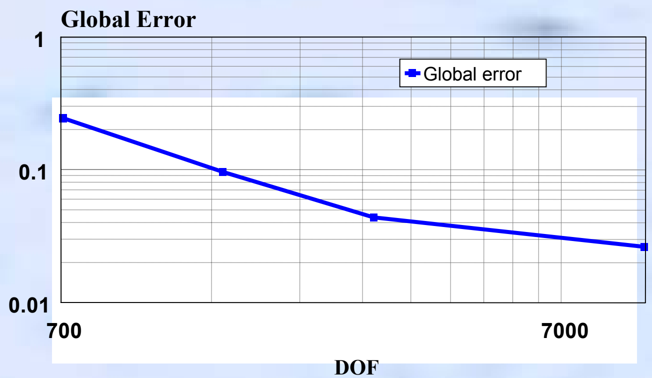


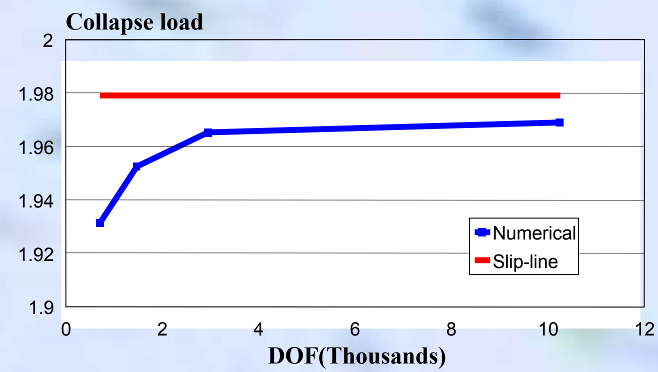
plastic multiplier field



Frictionless extrusion through a square die – Reduction 2/3

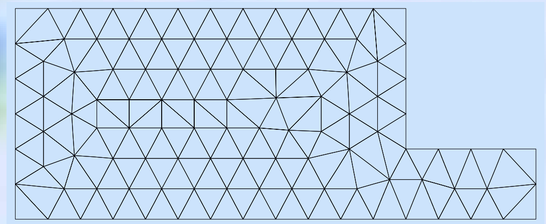


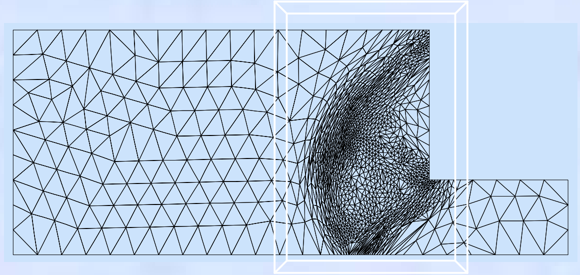


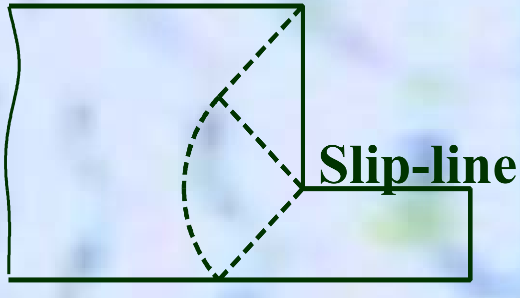


Initial - 707 DOF

After four Steps - 10255 DOF

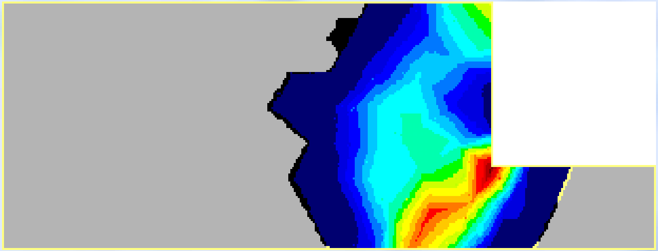


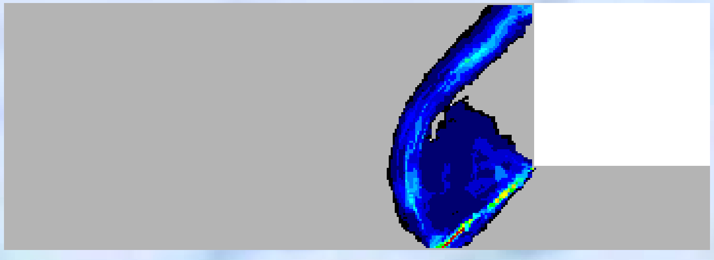




plastic multiplier field

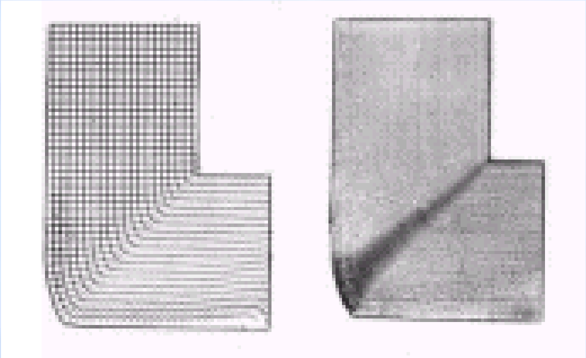
plastic multiplier field





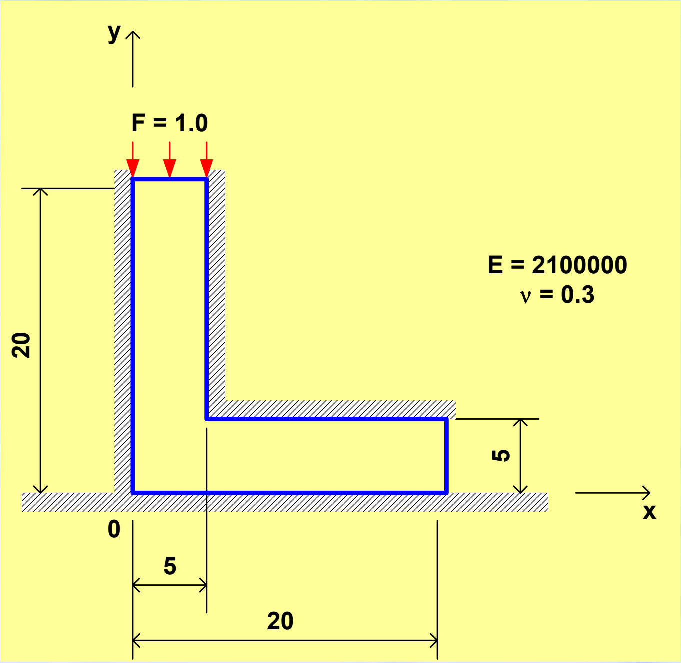
Equal channel angular extrusion (ECAE)

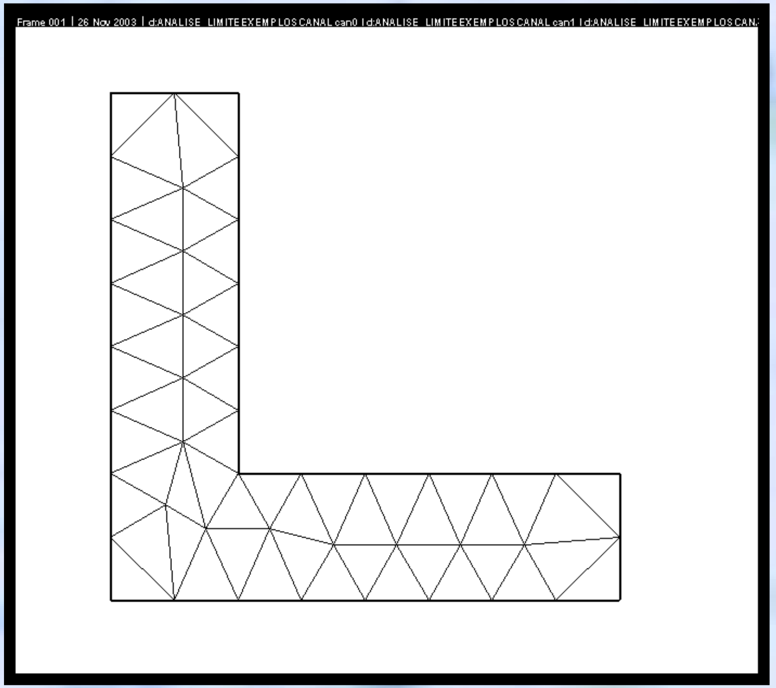
Method for deforming materials to very high plastic strains, with no net change in the billet's shape.



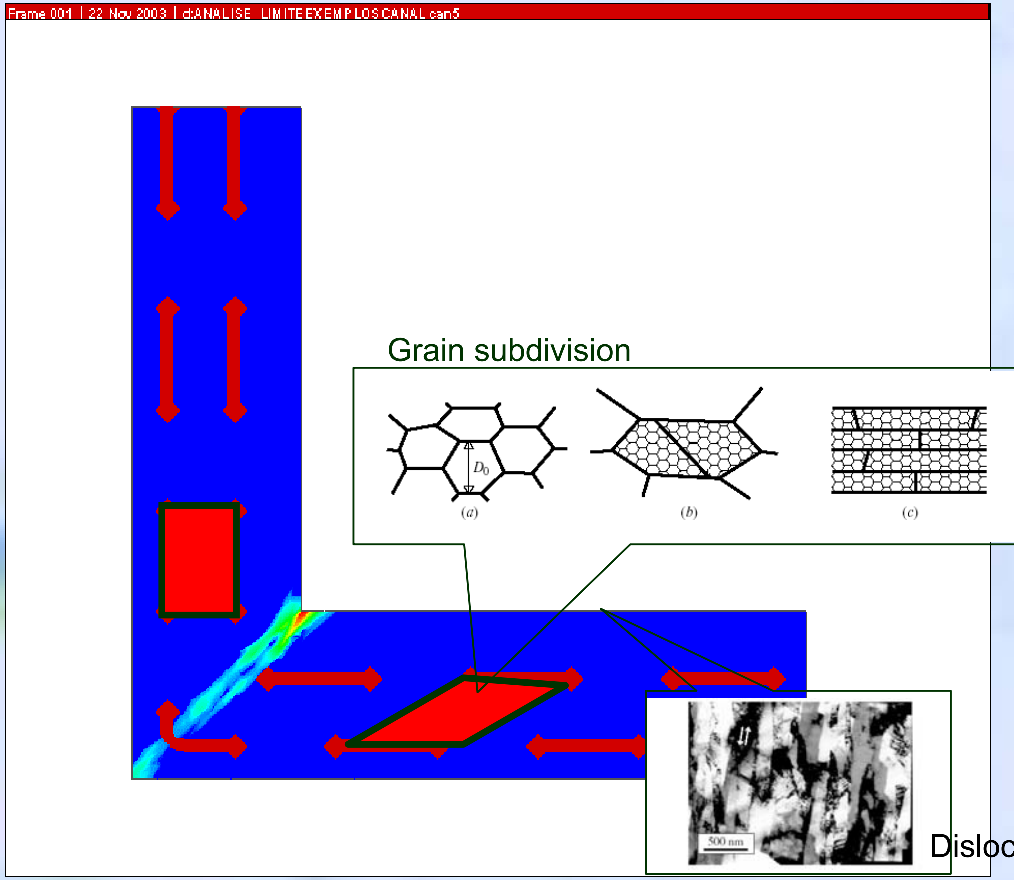
By grain refinement:

- control materials structure,
- texture and
- physico-mechanical properties.



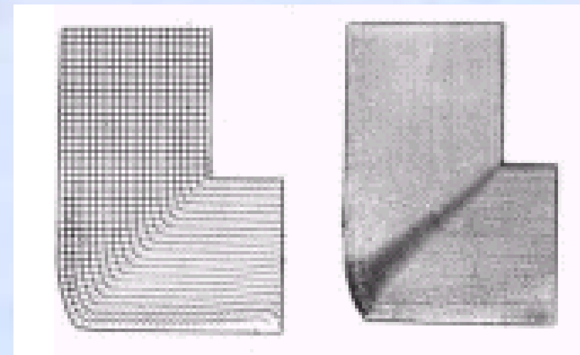


Nanostructured metals by severe plastic deformation



Dislocations accumulate in walls

Winther and Huang, 2003



Plastic Zone –Experimental
Segal (1999)

Plastic Zone and velocity field - Numerical

Limit States:

Basic direct methods for safety assessment of engineering structures subjected to variable loading.

Inelastic structures submitted to variable loads may undergo one of the following types of failure modes:

- **alternating plasticity (plastic shakedown),**
- **incremental collapse(ratcheting)**
- **instantaneous collapse (plastic collapse) - Limit analysis**

The objective in this analysis is the computation of the load amplification factor, α, for the domain of variable loadings, that ensures elastic shakedown.

Shakedown limits (Bree diagram)
for an Al_2O_3 / FGM / Ni 3-layer beam
$\rho_d = 0.6$
(1.2 mm FGM in 2 mm height of 3 layer beam)