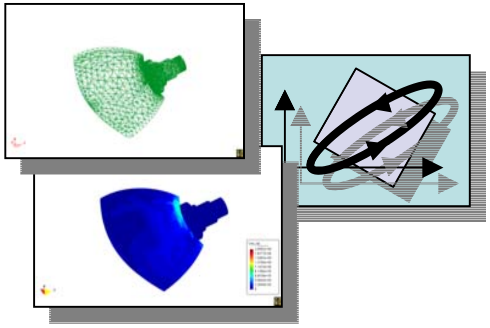
Fatigue Endurance under Multiaxial Loadings



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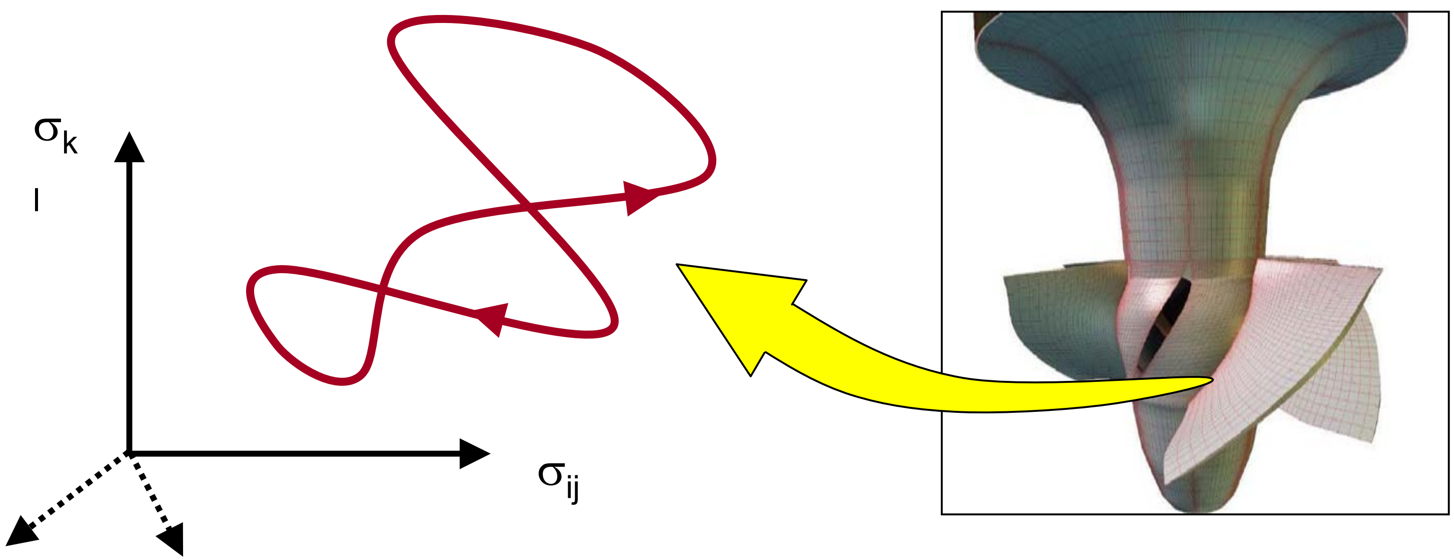
Goal:

To propose a fatigue model capable to answer the following question:

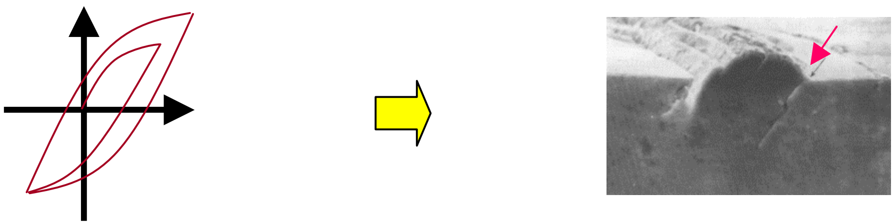
Under which conditions a structure subjected to dynamic multiaxial loads attains infinite number of cycles (> 10^6) without experiencing fatigue failure?



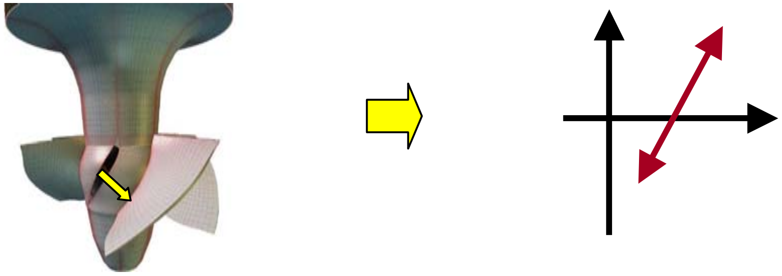
Phenomenological aspects:

In the setting of high cycle fatigue,

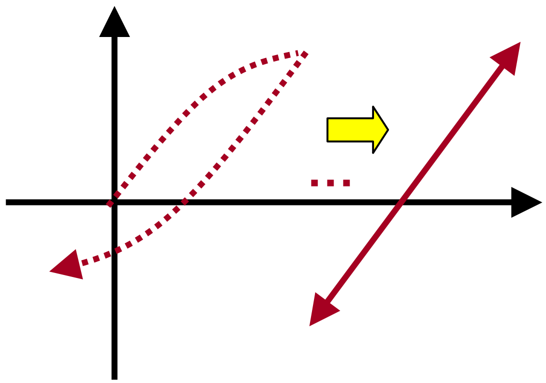
- mechanical degradation is mainly driven by localized plastic deformations at mesoscopic level,



- while the corresponding macroscopic behavior is essentially elastic:



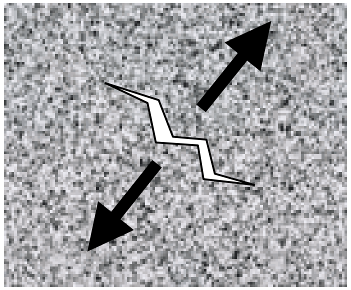
Thus, in order to avoid fatigue degradation, the mechanical behavior (at mesoscopic level) has to evolve to a state of elastic shakedown.



In metals, this can be accomplished only under certain bounded values of the "shear stress amplitude"

In our model: $\tau_{eq}(\mathbf{S})$ = appropriate function of the history of the deviatoric stress tensor $\mathbf{S}$ describing its "amplitude" in the multidimensional sense.

Tractive normal stresses also play an important role in solicitation to fatigue, by acting in mode I upon eventually pre-existing embryocracks in the material.



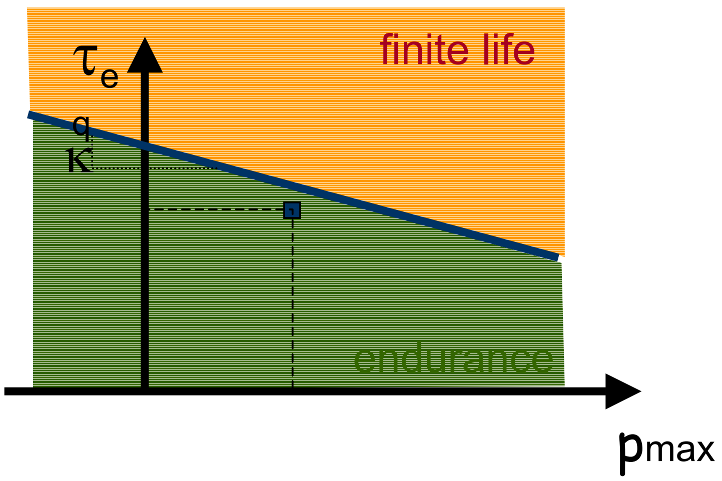
In our model:

$p_{\max}$ = maximum value of the hydrostatic stress p along the stress path.

(recalling that the hydrostatic stress is the average of the normal stress acting upon all the planes across a given material point)

Within this setting, let us write our fatigue endurance criterion as:

$$\tau_{eq}(\mathbf{S}) + \kappa \; p_{\max} \le \lambda$$

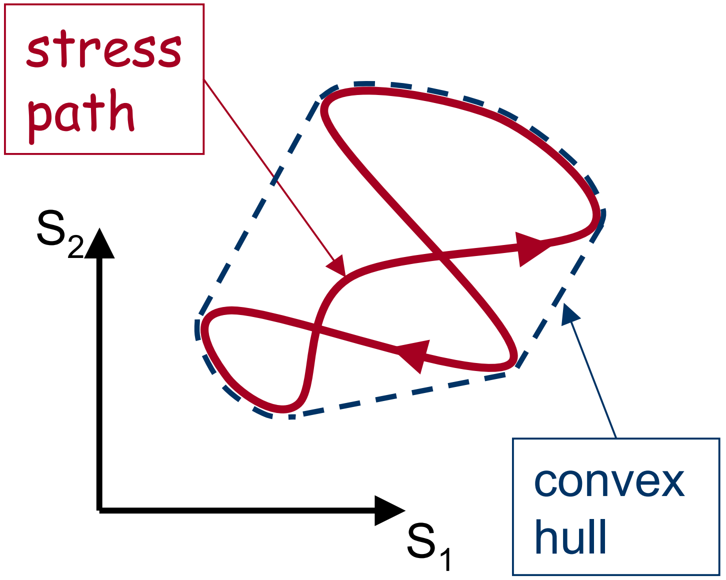


In what follows, we shall propose a measure of the shear stress amplitude τ_{eq} within the setting of multiaxial stress paths.

Shear stress amplitude:

Not all the states belonging to the stress path threatens the material point.

Only those states belonging to the corresponding convex hull determine the solicitation to fatigue.

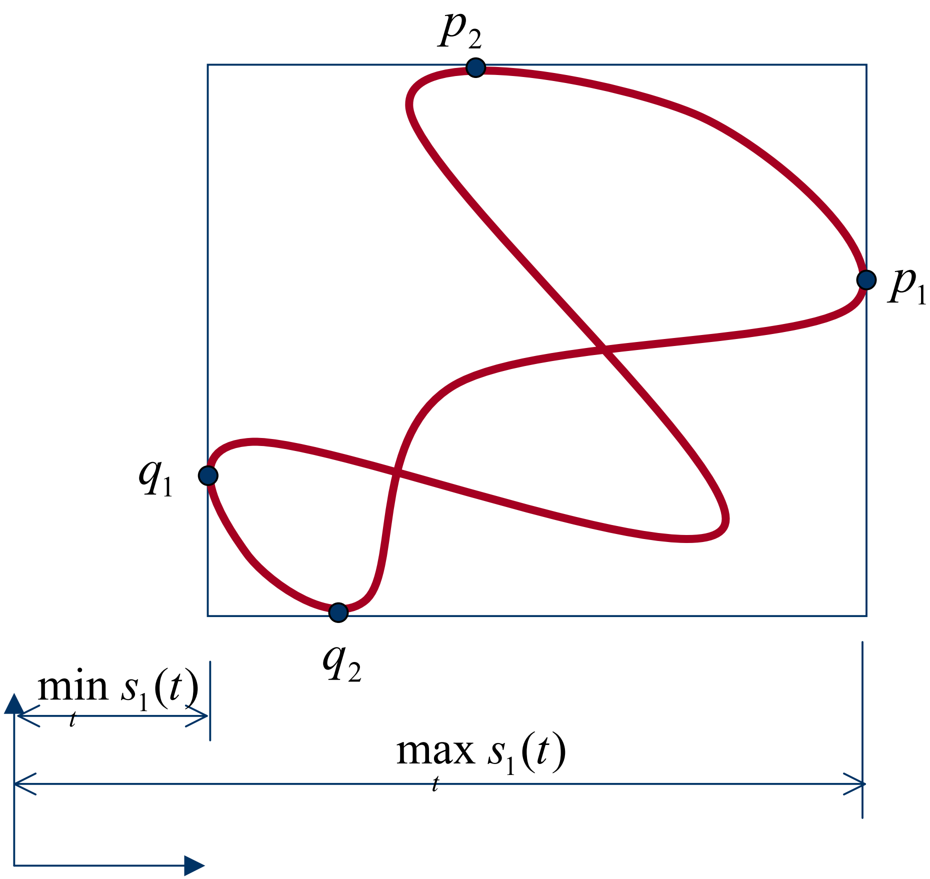


Shear stress amplitude can be defined from quantities associated with the convex hull.

The points of the stress path tangent to arbitrarily oriented prismatic hulls belong to the convex hull:

$$p_i = \arg(\max_t s_i(t)), \quad i = 1,\dots,5$$

$$q_i = \arg(\min_t s_i(t)), \quad i = 1,\dots,5$$



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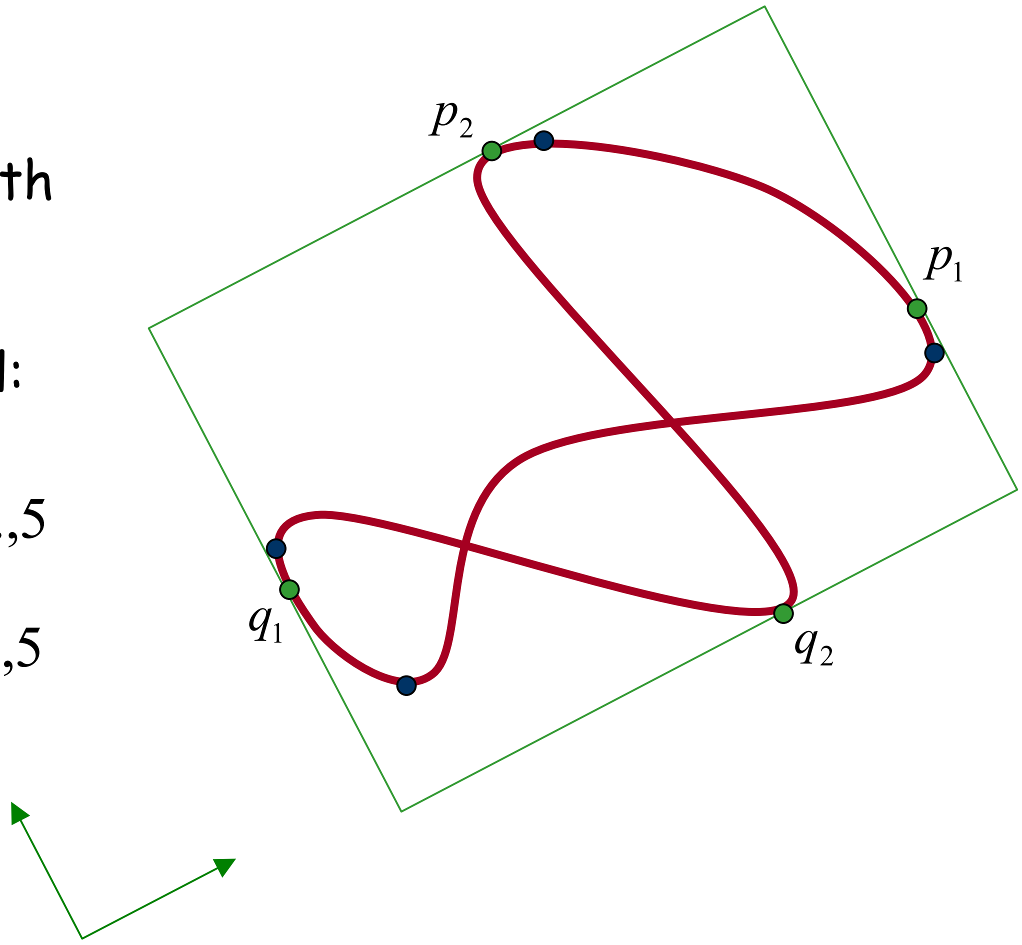

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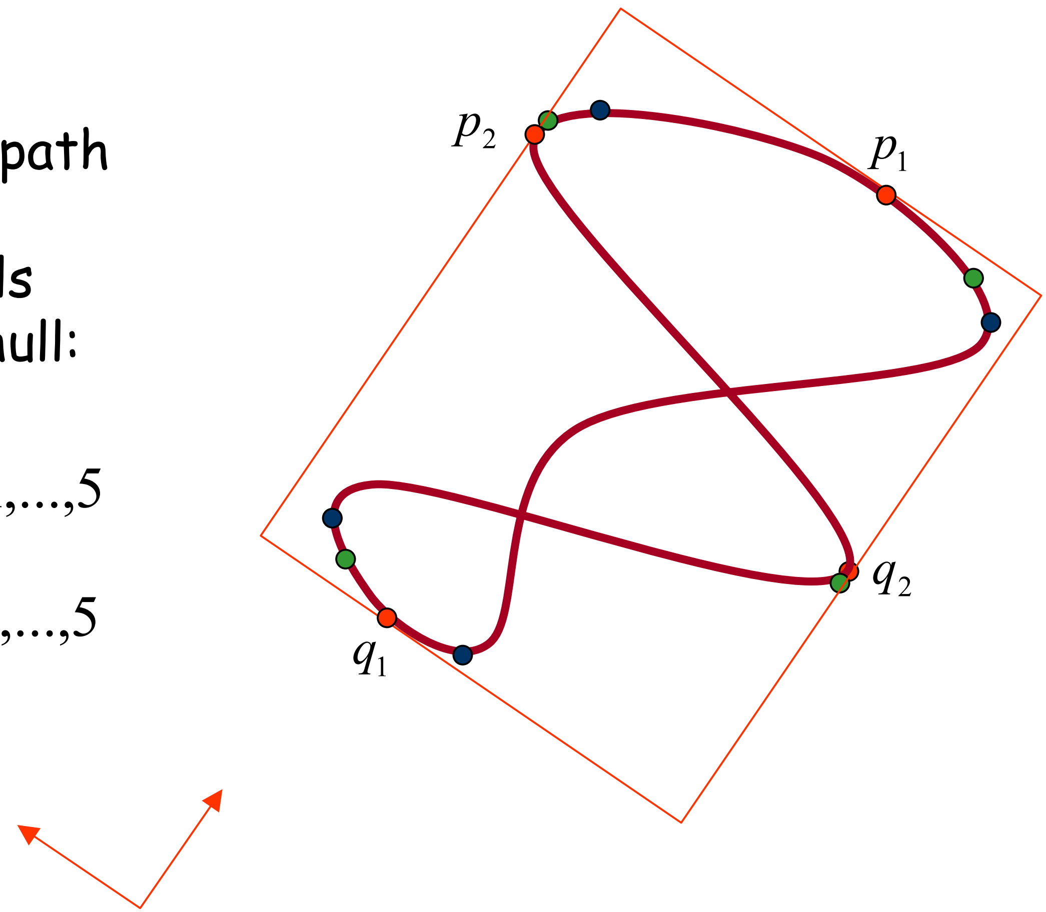
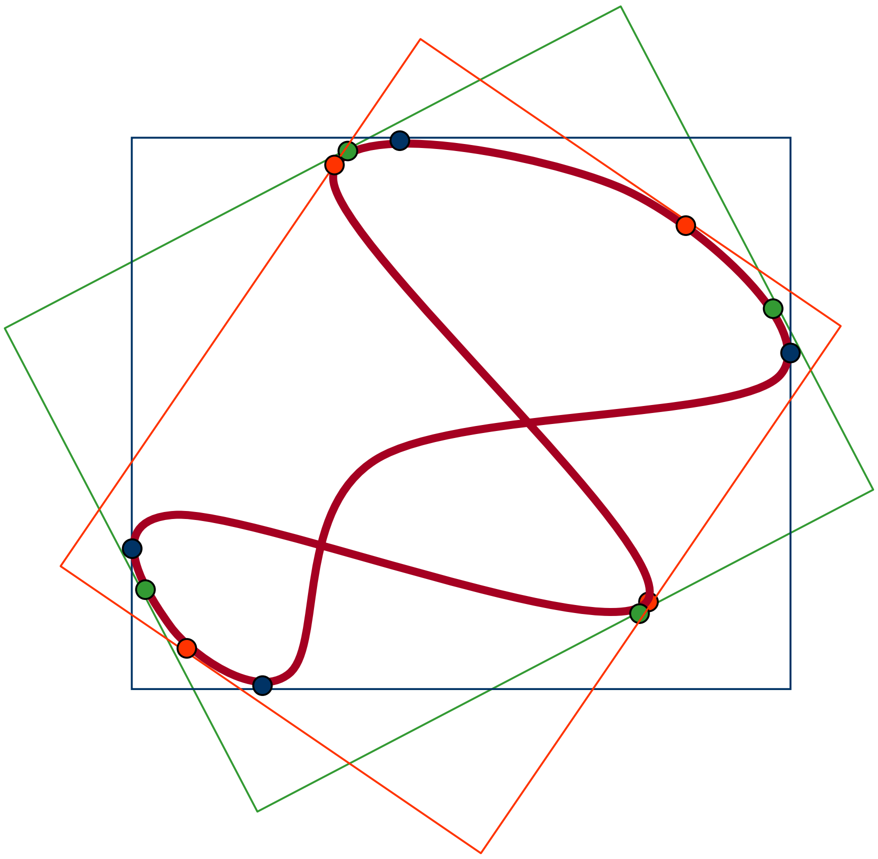


As a consequence, the set of prismatic hulls itself and its corresponding quantities:

$$\max_t s_i(t), \quad \min_t s_i(t), \quad i = 1,\ldots,5$$

can be considered for the characterization of the convex hull

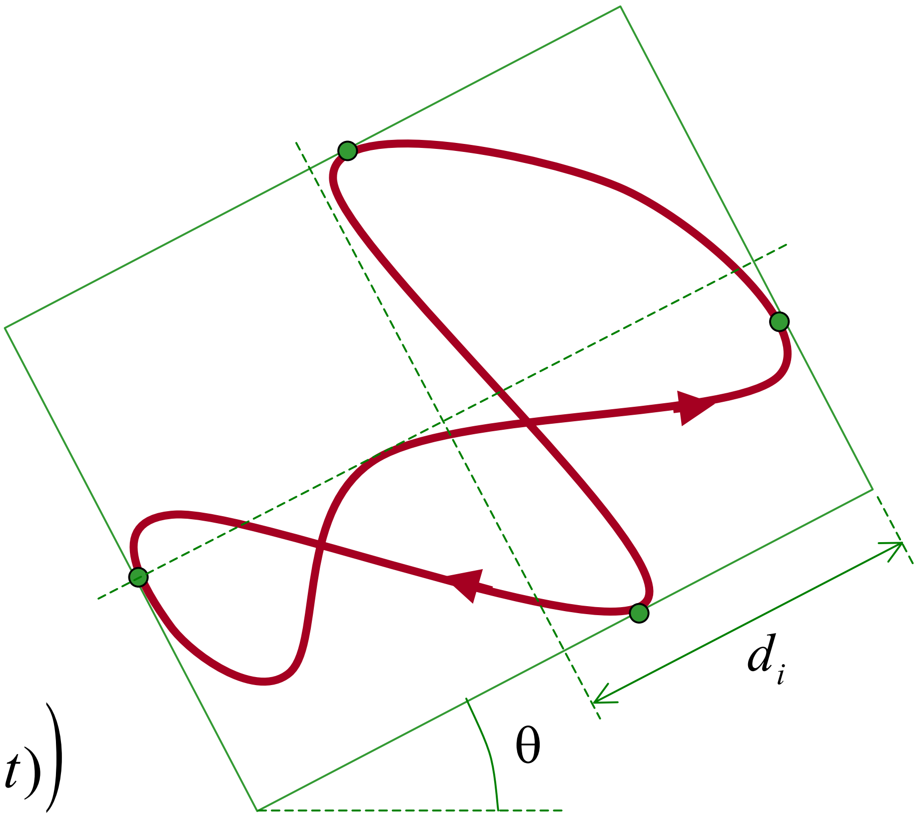


We consider the following quantity as a measure of the shear stress amplitude:

$$\tau_{eq} = \sum_i \left(d_i^2\right)^{1/2}$$

where:

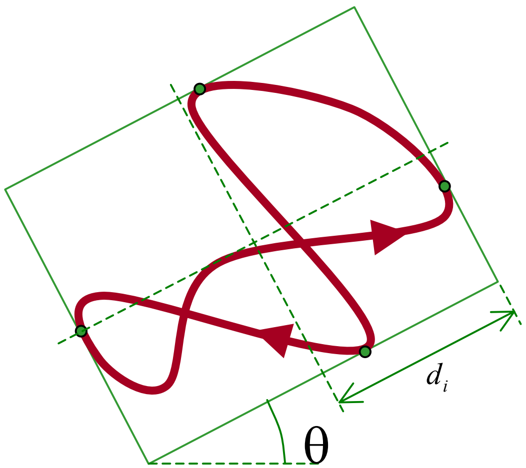
$$d_i = \frac{1}{2}\max_{\theta}\left(\max_t s_i(\theta;t) - \min_t s_i(\theta;t)\right)$$

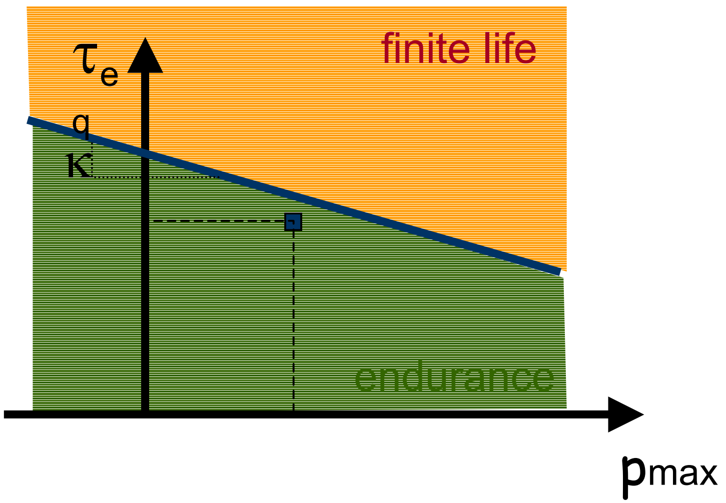


Remark: θ is the orientation of the prismatic hull in the 5-dimensional space of deviatoric stresses

The resulting fatigue endurance criterion is hence given by:

$$\sqrt{\sum_{i=1}^{5} d_i^2} + \kappa \; p_{\max} \leq \lambda \qquad \text{where:} \quad d_i = \frac{1}{2} \max_{\theta} \left(\max_{t} s_i(\theta;t) - \min_{t} s_i(\theta;t) \right)$$





Computational issues

- The search for the orientation of the prismatic hull which gives the global maximum value of:

$$\tau(\theta) = \sum_{i=1}^{5} d_i^2$$

is performed in the 5-dimensional deviatoric space. Jacobi (or Givens) rotations were considered for simplicity. On the other hand, this implies a 10-parametric rotation process.

- The function $\tau(\theta)$ may attain several local maxima and hence some care must be taken with respect to the maximization algorithm.

Assessment

Proportional and nonproportional multiaxial fatigue experiments for different materials were considered to assess the proposed criterion in predicting fatigue strength under a high number of cycles.

Limiting situations of fatigue endurance reported by:

set	authors	Material	f_{-1}	t_{-1}
1	Nihihara & Kawamoto (1945)	hard steel	313.9	196.2
2	Heindereich, Zenner & Richter (1983)	34Cr4	410	256
3	Heindereich, Zenner & Richter (1983)	34Cr4	415	256
4	Kaniut (1983)	25CrMo4	340	228
5	Mielke (1980)	25CrMo4	340	228

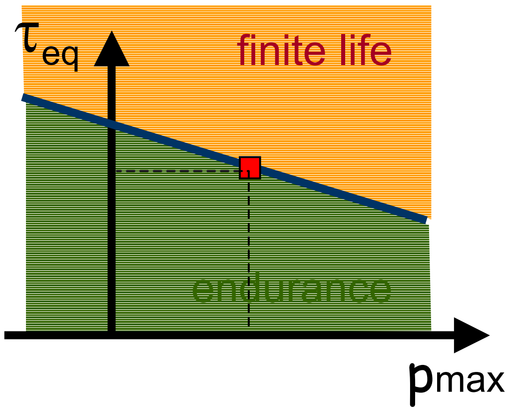
Error index: evaluation of limiting situations

$$\tau_{eq} + \kappa\ p_{\max} \leq \lambda$$ ← fatigue endurance criterion

$$I = \frac{\tau_{eq} + \kappa\ p_{\max} - \lambda}{\lambda} \times 100 \leq 0$$ ← error index

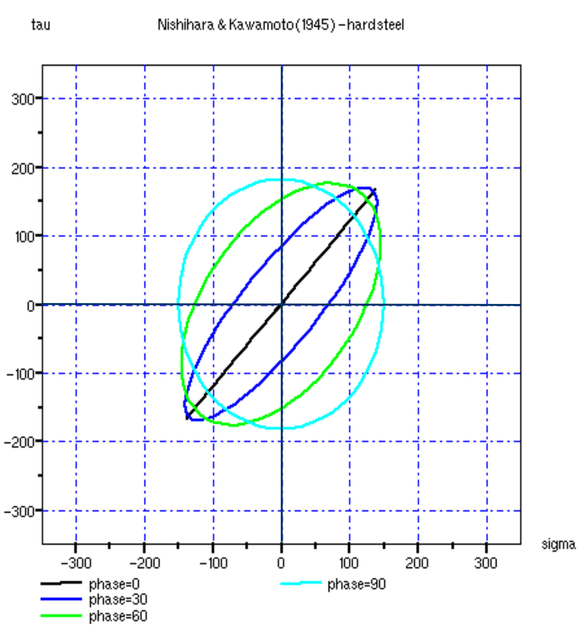
$I > 0$ → conservative prediction

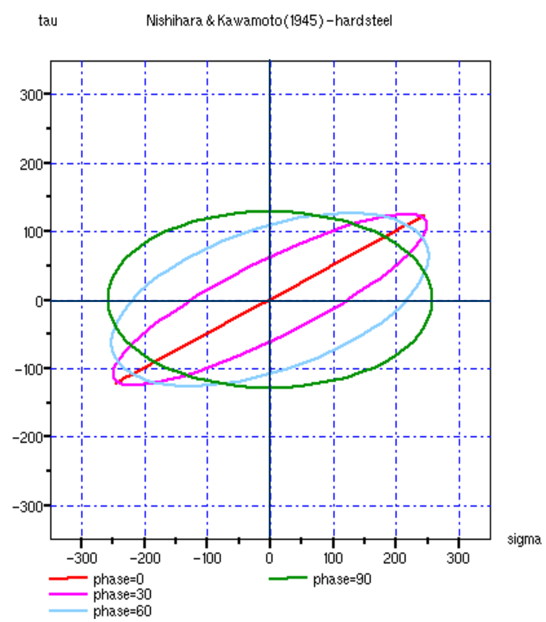
$I < 0$ → non-conservative

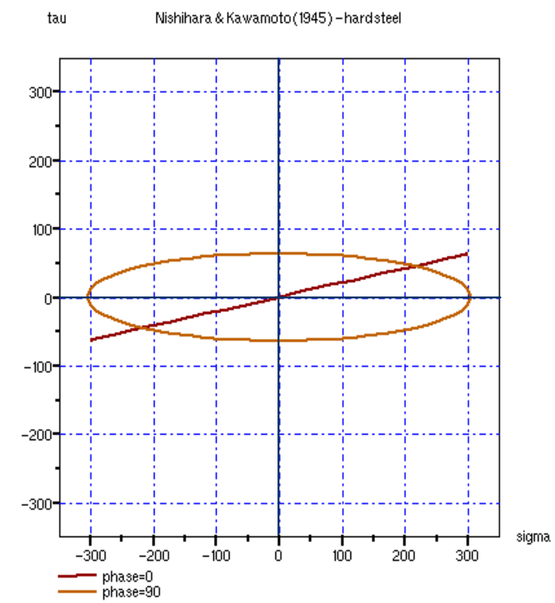


Nishihara & Kawamoto (1945), hard steel

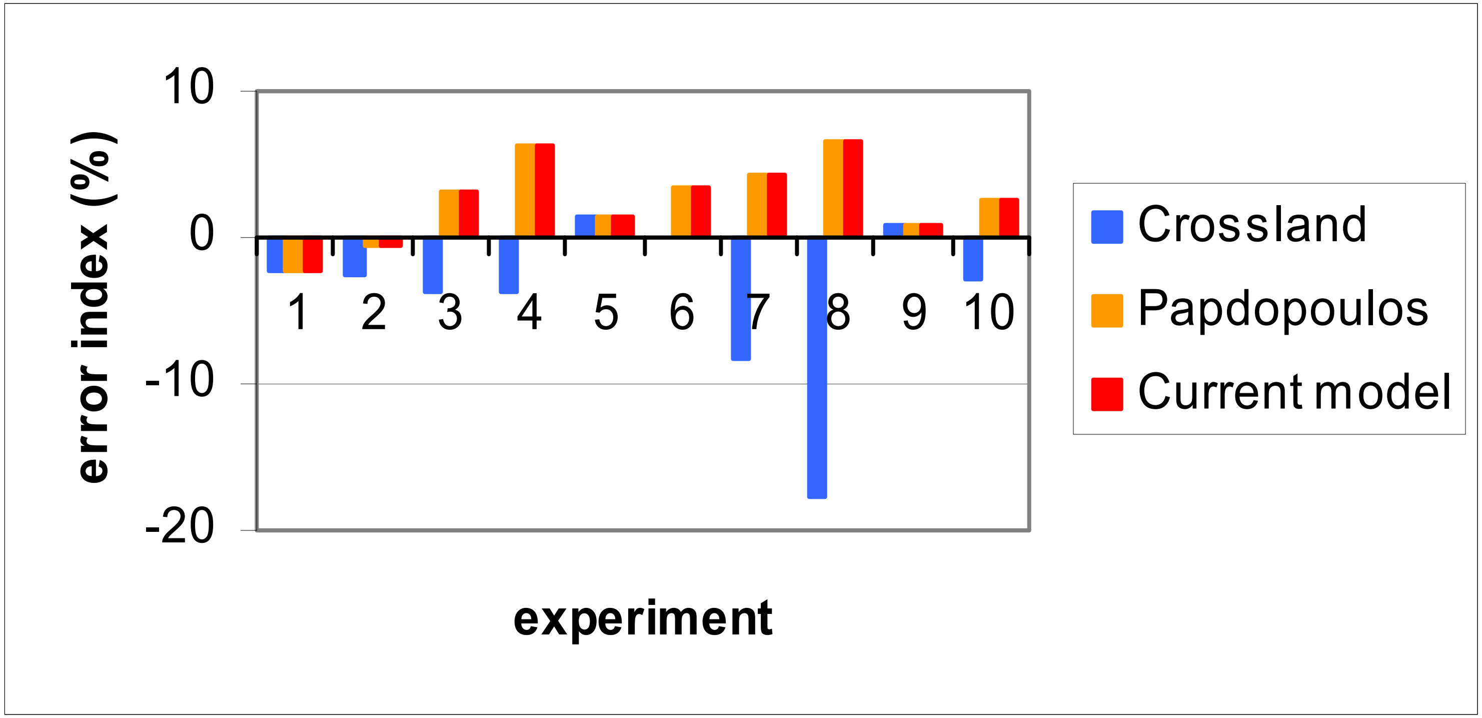
Proportional and nonproportional $\sigma-\tau$,
same frequency of excitation, no mean stress







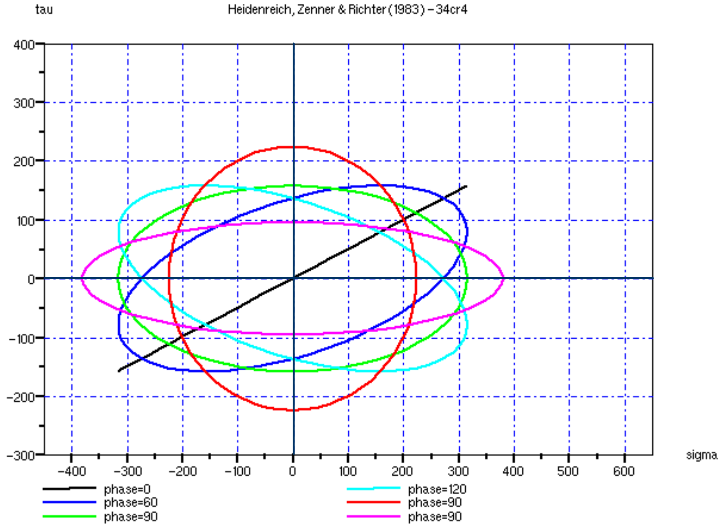
Nishihara & Kawamoto (1945), hard steel

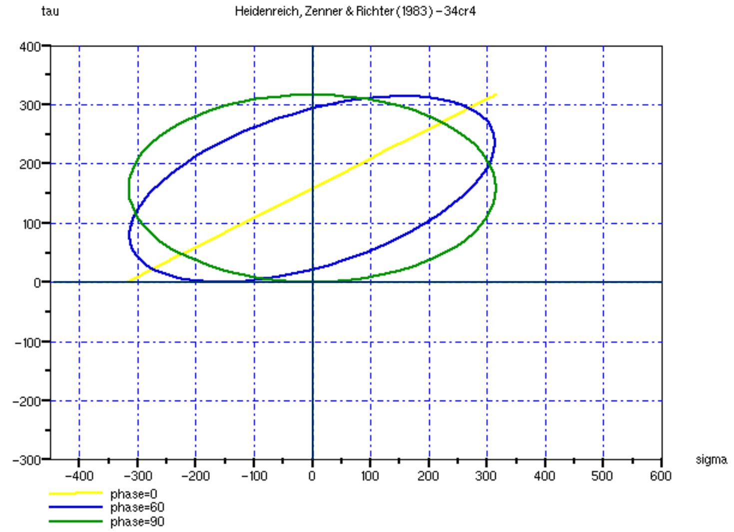


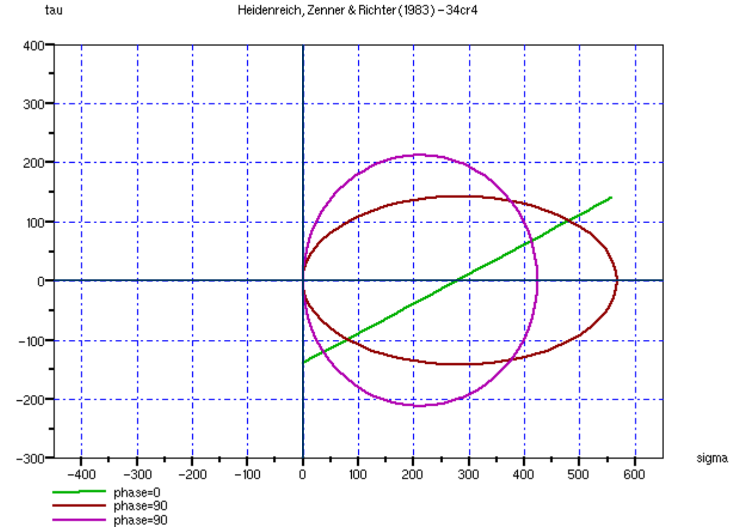
-2.3% < I < 6.5% (current model)

Heindereich, Zenner & Richter (1983), 34Cr4

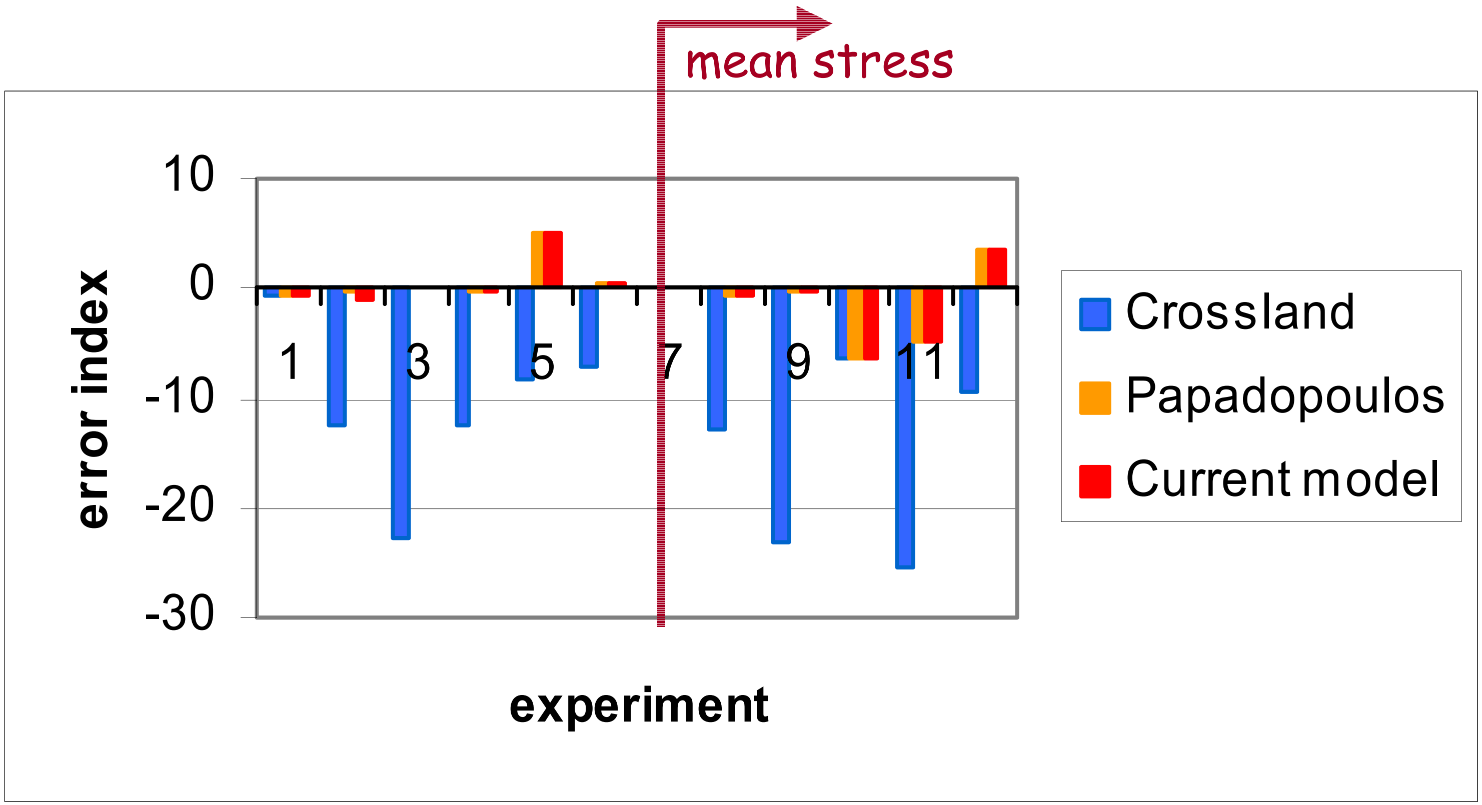
Proportional and nonproportional $\sigma-\tau$, same frequency of excitation







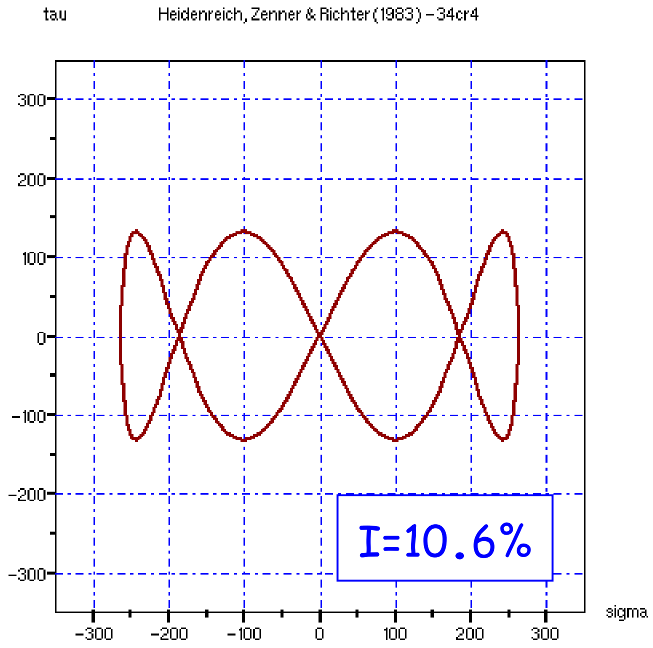
Heindereich, Zenner & Richter (1983), 34Cr4



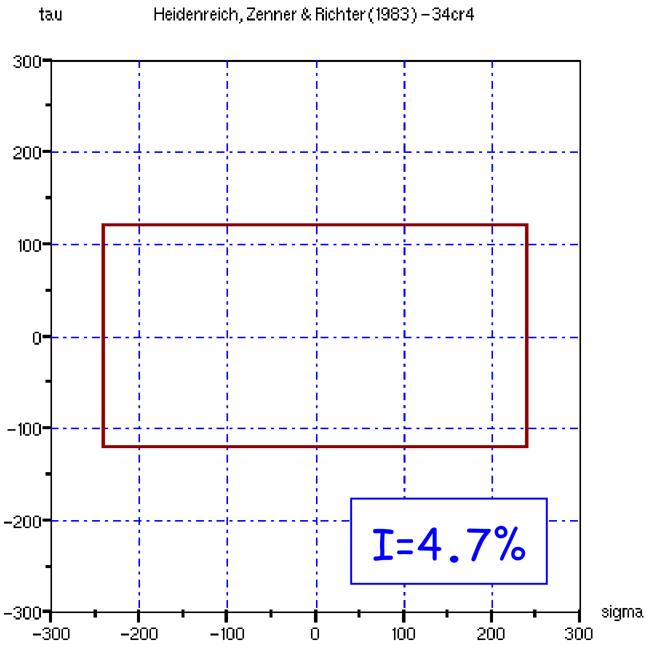
-6.4% < I < 5.2%   (current model)

Heindereich, Zenner & Richter (1983), 34Cr4

Nonproportional $\sigma-\tau$, $w_\tau = 4\, w_\sigma$

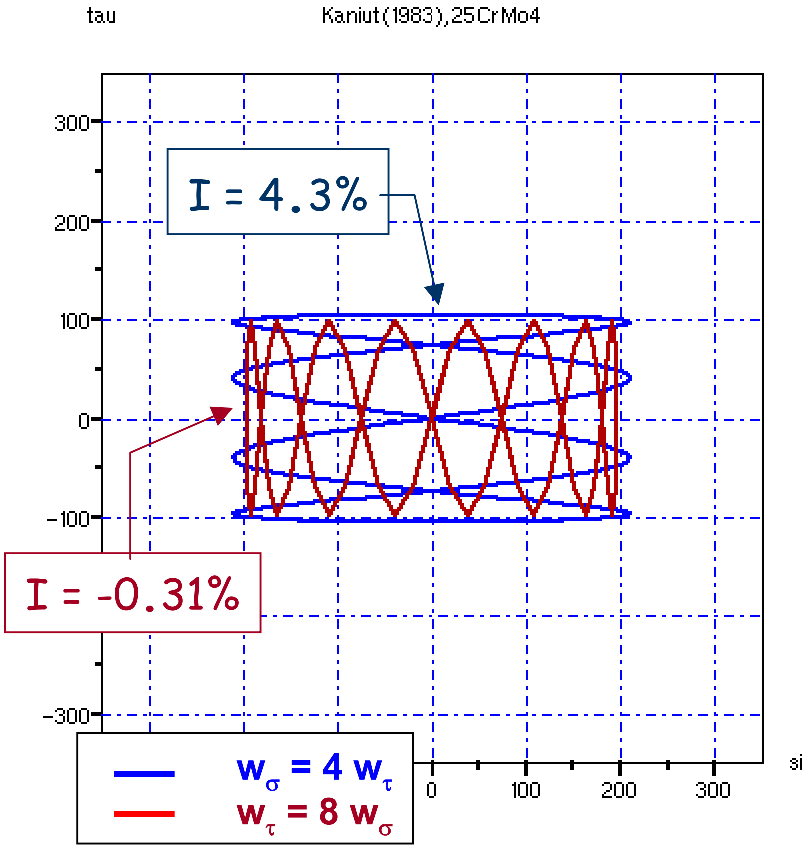


Piecewise linear $\sigma-\tau$

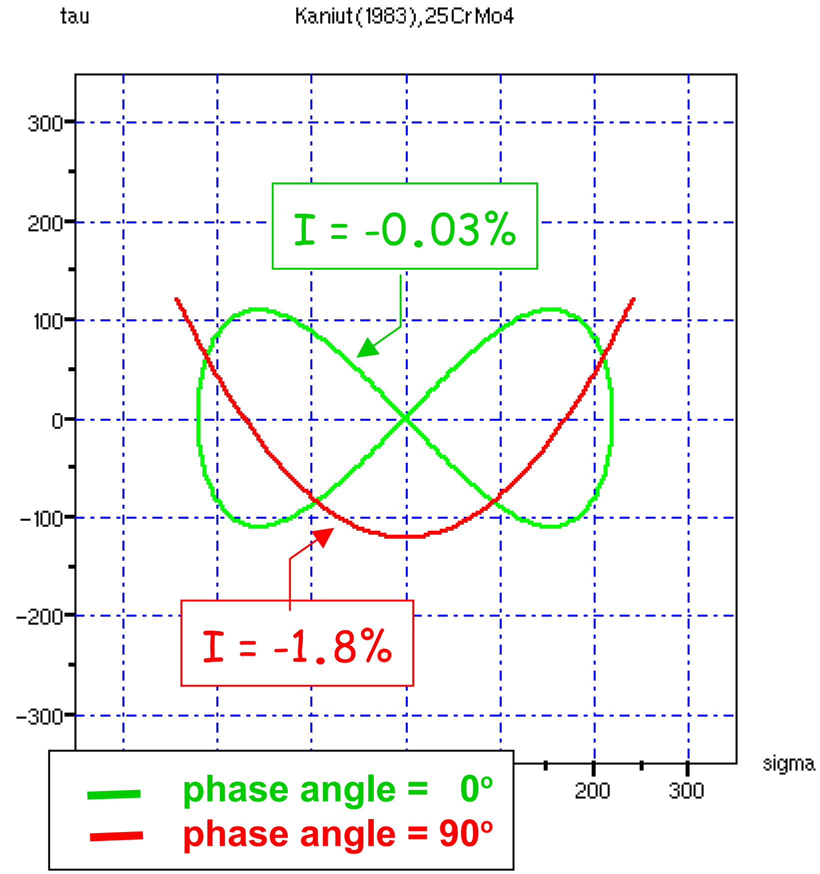


Kaniut (1983), 25CrMo4

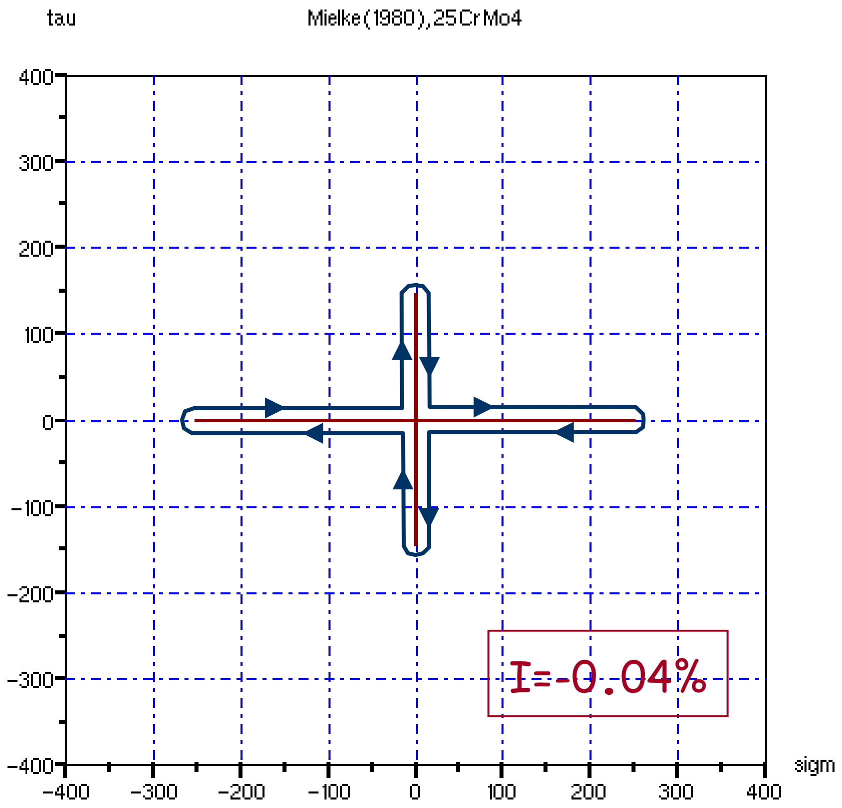
Nonproportional σ–τ

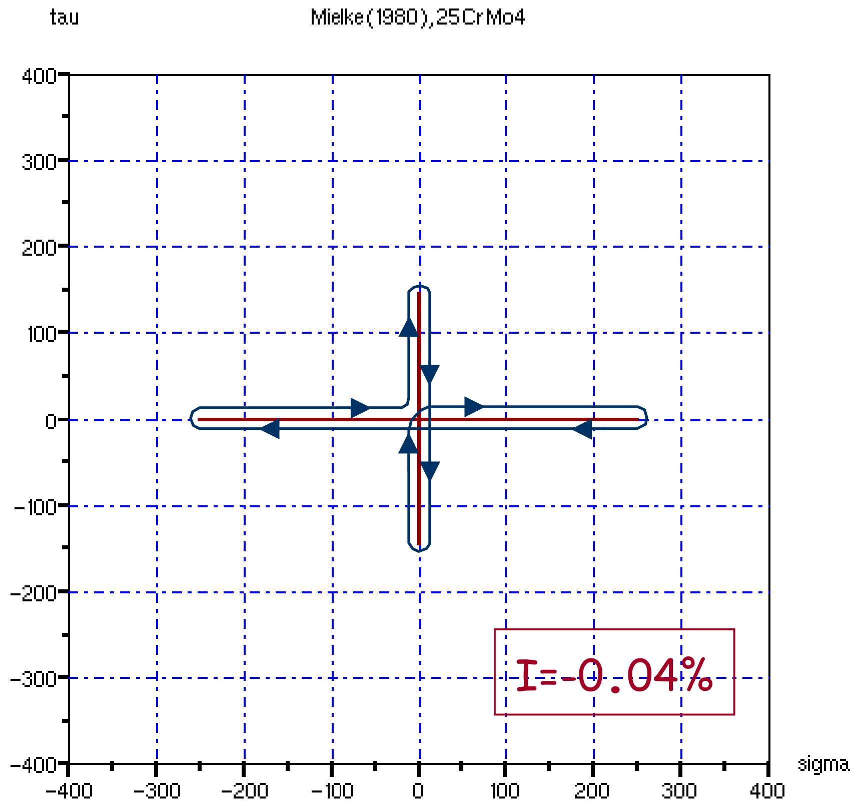


Nonproportional σ–τ, $w_\tau = 2\ w_\sigma$

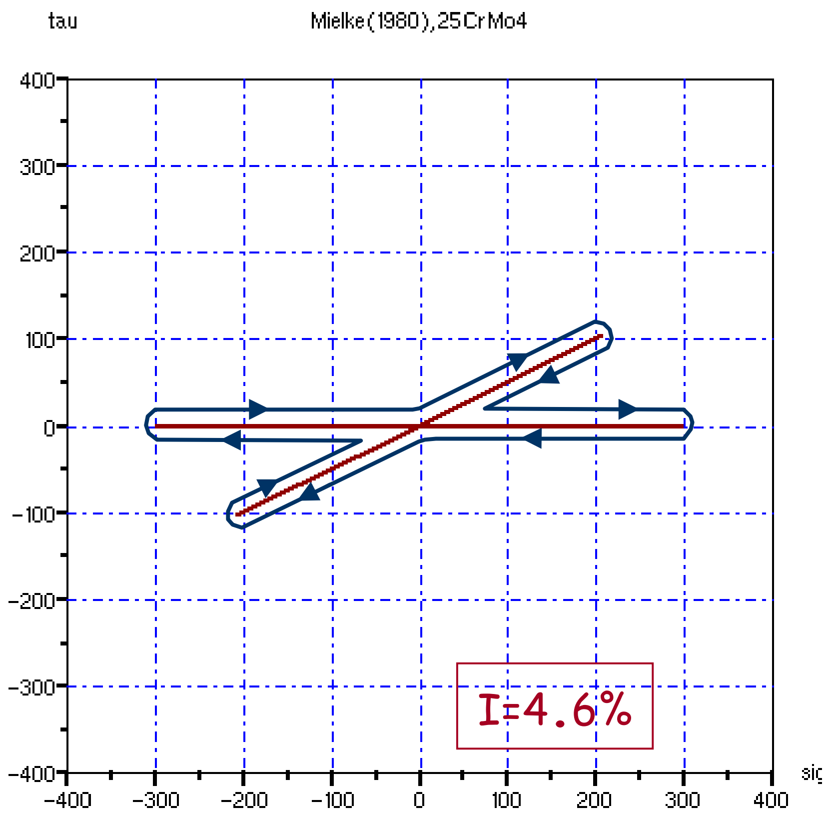


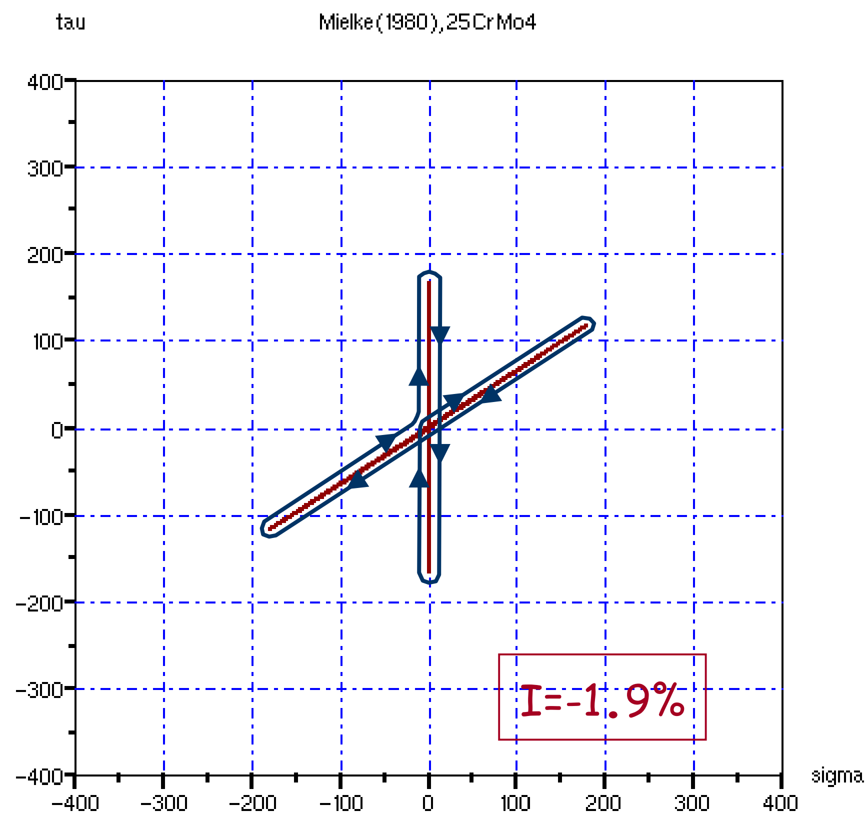
Mielke (1980), 25CrMo4





Mielke (1980), 25CrMo4





Closure

- A new stress based multiaxial fatigue criterion, which is very simple to implement and can be applied to a broad class of loadings, has been proposed;
- Application of the proposed criterion for several different materials yielded very good predictions of fatigue endurance;
- We are conducting studies in order to extend the applicability of the criterion to more ductile materials.
- We are also addressing the question of fatigue endurance under conditions of severe stress gradients.

Thank you !!!