The Ground Structure Method

A computational method for optimal frames (pin-jointed frames)

TOP Gang

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Presentation: 11/7/2016

References (General)

- **General references: Elastic and Plastic Formulations**

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References (Plastic Analysis)

- **Ground Structure Method**

  Dorn, WS; Gomory, RE; Greenberg, HJ "Automatic design of optimal structures", Journal de mècanique 3(1), pp. 25-52, (1964)

- **Relevant References**

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  Karmarkar, N “A new polynomial-time algorithm for linear programming”, Combinatorica 4(4), pp. 373-395, (1984)

References (Plastic Analysis)

- **Relevant References**

  Zegard, T; Paulino, GH "GRAND — Ground structure based topology optimization for arbitrary 2D domains using MATLAB“, Structural and Multidisciplinary Optimization 50(5), pp. 861-882, (2014)

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References (Elastic Analysis)

- **Relevant References**

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- **Relevant References**

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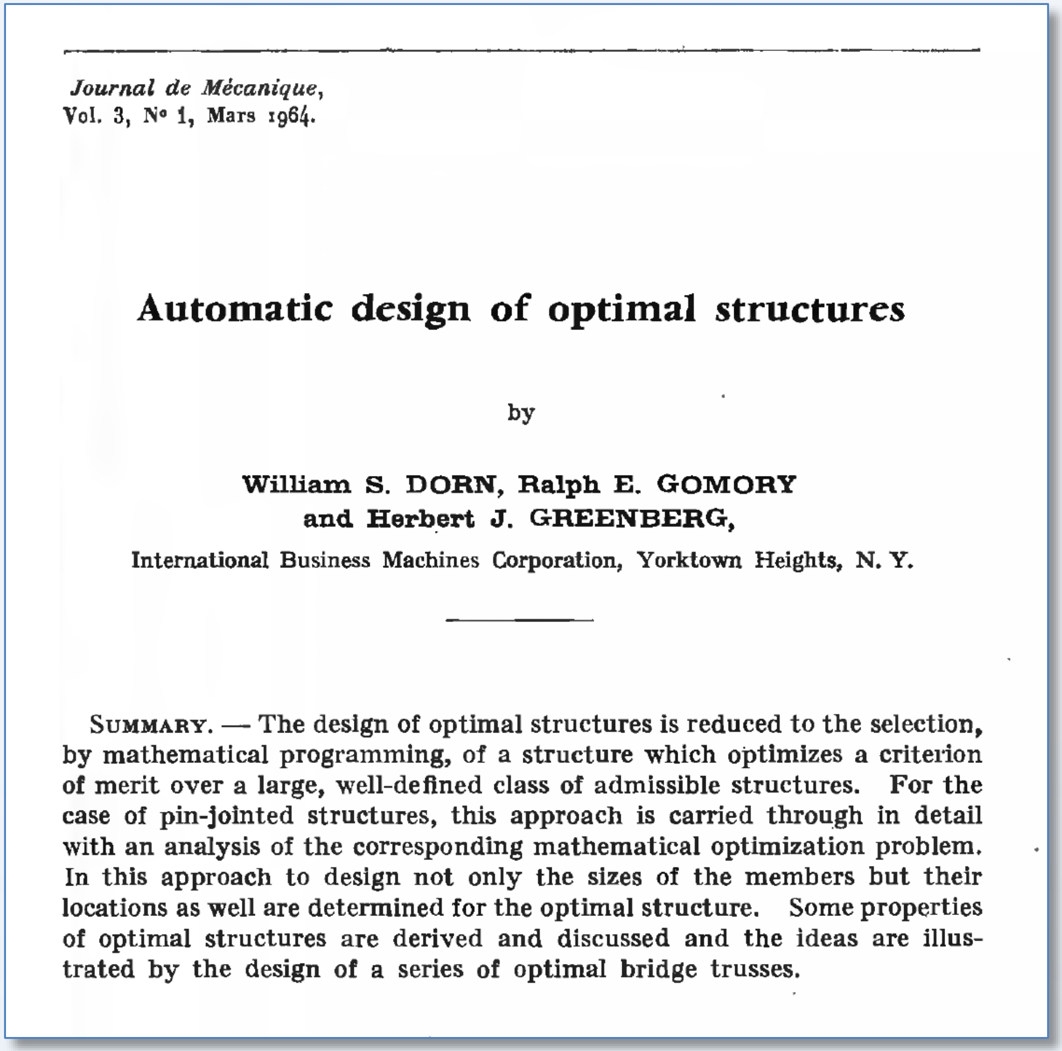
PLASTIC FORMULATION

References & Information

- Journal de Mècanique



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Automatic design of optimal structures

by

William S. DORN, Ralph E. GOMORY
and Herbert J. GREENBERG,
International Business Machines Corporation, Yorktown Heights, N. Y.

Summary. — The design of optimal structures is reduced to the selection, by mathematical programming, of a structure which optimizes a criterion of merit over a large, well-defined class of admissible structures. For the case of pin-jointed structures, this approach is carried through in detail with an analysis of the corresponding mathematical optimization problem. In this approach to design not only the sizes of the members but their locations as well are determined for the optimal structure. Some properties of optimal structures are derived and discussed and the ideas are illustrated by the design of a series of optimal bridge trusses.

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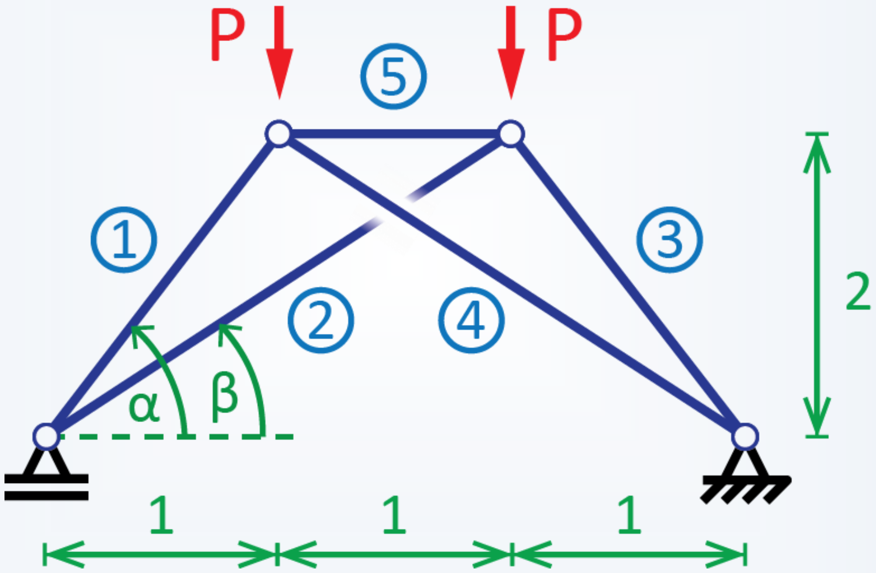
Force Equilibrium Matrix

- Given the following problem

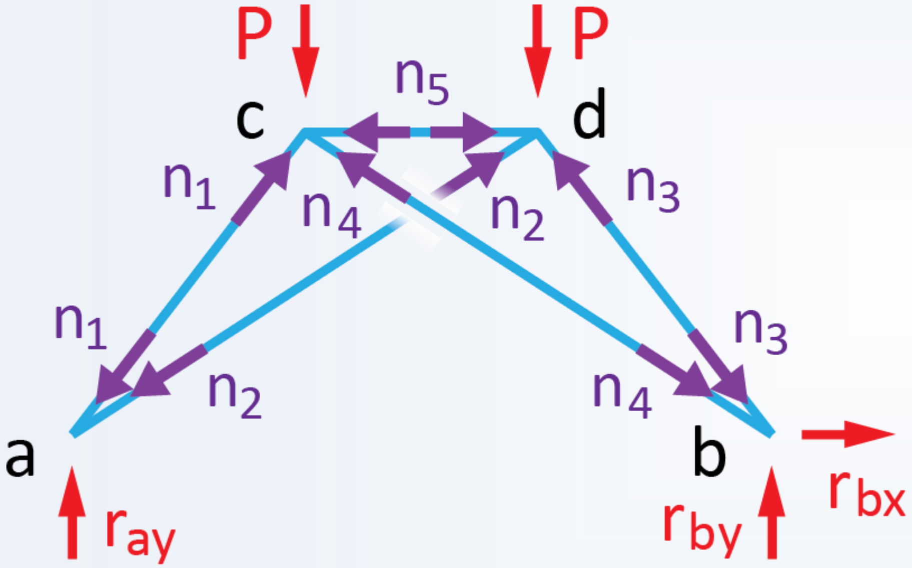


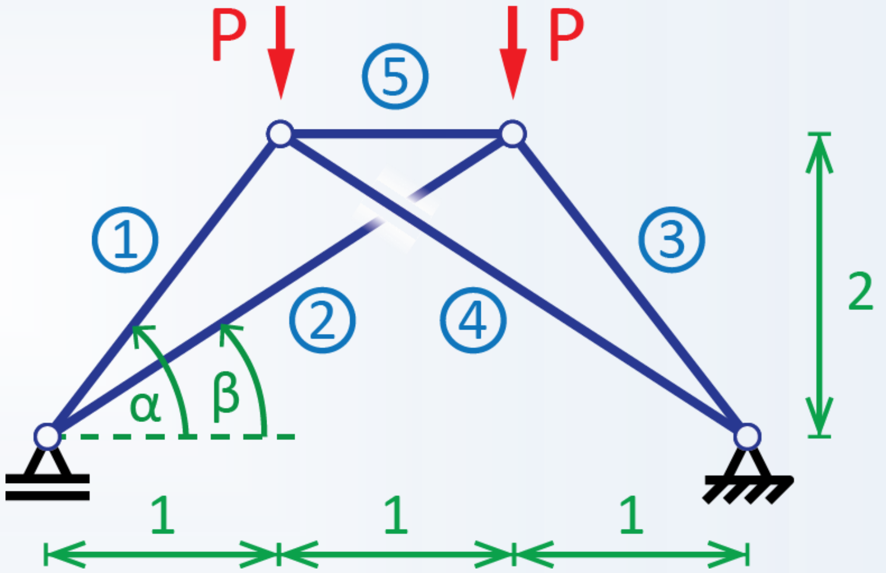
$$\cos\alpha = 1/\sqrt{5} \qquad \sin\alpha = 2/\sqrt{5}$$

$$\cos\beta = 1/\sqrt{2} \qquad \sin\beta = 1/\sqrt{2}$$

Force Equilibrium Matrix

- Equations for nodal equilibrium

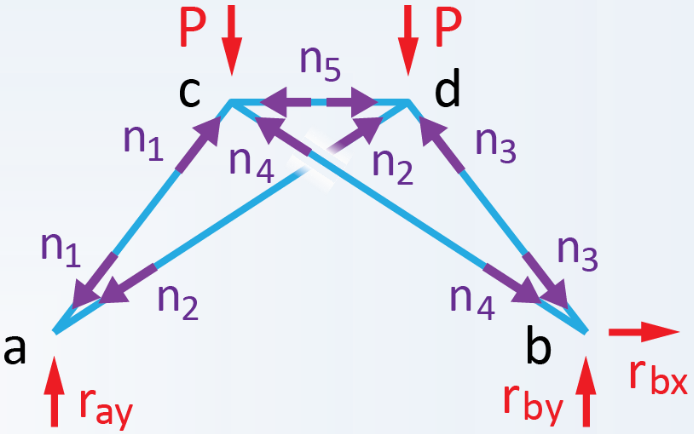




$$\sum F_{ax} \;:\; -n_1 \cos\alpha - n_2 \cos\beta = 0$$
$$\sum F_{ay} \;:\; -n_1 \sin\alpha - n_2 \sin\beta = r_{ay}$$
$$\sum F_{bx} \;:\; n_3 \cos\alpha + n_4 \cos\beta = r_{bx}$$
$$\sum F_{by} \;:\; -n_3 \sin\alpha\, N_3 - n_4 \sin\beta = r_{by}$$

Force Equilibrium Matrix

- Equations for nodal equilibrium



Taking: $\cos\alpha \rightarrow c_\alpha \quad \sin\alpha \rightarrow s_\alpha$

$\cos\beta \rightarrow c_\beta \quad \sin\beta \rightarrow s_\beta$

$$\begin{bmatrix} -c_\alpha & -c_\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & c_\alpha & c_\beta & 0 & 0 & -1 & 0 \\ 0 & 0 & -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 \\ c_\alpha & 0 & 0 & -c_\beta & -1 & 0 & 0 & 0 \\ s_\alpha & 0 & 0 & s_\beta & 0 & 0 & 0 & 0 \\ 0 & c_\beta & -c_\alpha & 0 & 1 & 0 & 0 & 0 \\ 0 & s_\beta & s_\alpha & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ r_{ay} \\ r_{bx} \\ r_{by} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ -P \end{Bmatrix}$$

Force Equilibrium Matrix

- Equations for nodal equilibrium

$$\begin{bmatrix} \mathbf{B} & \mathbf{B_{nr}} \\ \mathbf{B_{rn}} & \mathbf{B_{rr}} \end{bmatrix}^{\mathbf{T}} \begin{Bmatrix} \mathbf{n} \\ \mathbf{r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

Matrix is **square** and **invertible**:
System is statically determinate

$$\begin{bmatrix} -c_\alpha & -c_\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & c_\alpha & c_\beta & 0 & 0 & -1 & 0 \\ 0 & 0 & -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 \\ c_\alpha & 0 & 0 & -c_\beta & -1 & 0 & 0 & 0 \\ s_\alpha & 0 & 0 & s_\beta & 0 & 0 & 0 & 0 \\ 0 & c_\beta & -c_\alpha & 0 & 1 & 0 & 0 & 0 \\ 0 & s_\beta & s_\alpha & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ r_{ay} \\ r_{bx} \\ r_{by} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ -P \end{Bmatrix}$$

Force Equilibrium Matrix

- Equations for nodal equilibrium

$$\begin{bmatrix} \mathbf{B} & \mathbf{B_{nr}} \\ \mathbf{B_{rn}} & \mathbf{B_{rr}} \end{bmatrix}^{\mathbf{T}} \begin{Bmatrix} \mathbf{n} \\ \mathbf{r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

To solve for **n**, we only need:

$$\mathbf{B}^{\mathbf{T}}\mathbf{n} = \mathbf{f}$$

$$\begin{bmatrix} -c_\alpha & -c_\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & c_\alpha & c_\beta & 0 & 0 & -1 & 0 \\ 0 & 0 & -s_\alpha & -s_\beta & 0 & 0 & 0 & -1 \\ c_\alpha & 0 & 0 & -c_\beta & -1 & 0 & 0 & 0 \\ s_\alpha & 0 & 0 & s_\beta & 0 & 0 & 0 & 0 \\ 0 & c_\beta & -c_\alpha & 0 & 1 & 0 & 0 & 0 \\ 0 & s_\beta & s_\alpha & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ r_{ay} \\ r_{bx} \\ r_{by} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ -P \end{Bmatrix}$$

Force Equilibrium Matrix

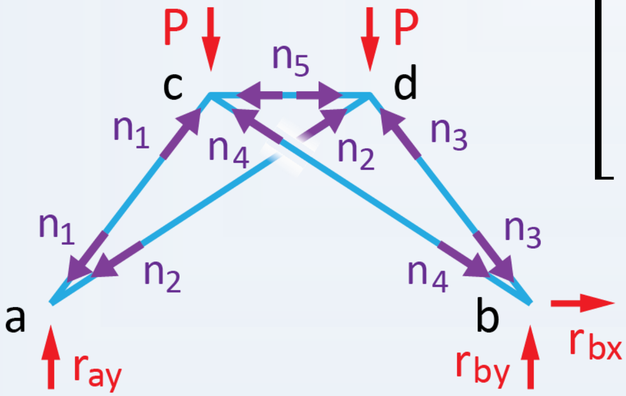
- Equations for nodal equilibrium

$$\begin{bmatrix} \mathbf{B} & \mathbf{B_{nr}} \\ \mathbf{B_{rn}} & \mathbf{B_{rr}} \end{bmatrix}^{\mathbf{T}} \begin{Bmatrix} \mathbf{n} \\ \mathbf{r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

To solve for **n**, we only need:

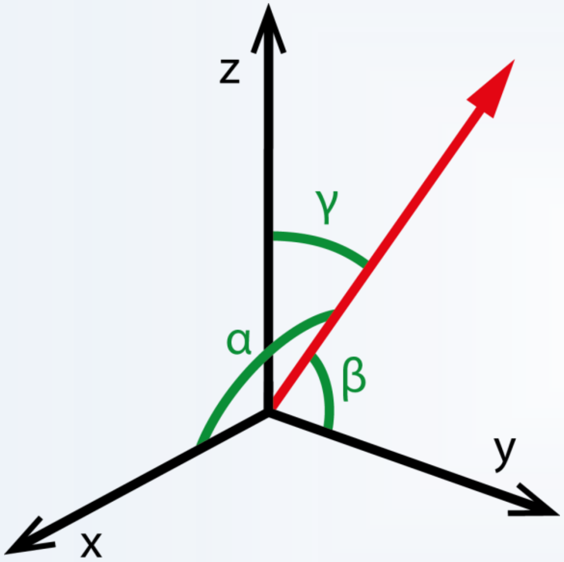
$$\mathbf{B^T n = f}$$

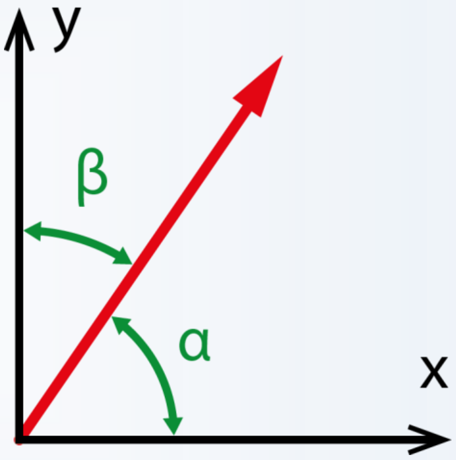
$$\begin{bmatrix} -c_\alpha & -c_\beta & 0 & 0 & 0 \\ c_\alpha & 0 & 0 & -c_\beta & -1 \\ s_\alpha & 0 & 0 & s_\beta & 0 \\ 0 & c_\beta & -c_\alpha & 0 & 1 \\ 0 & s_\beta & s_\alpha & 0 & 0 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -P \\ 0 \\ -P \end{Bmatrix}$$



Directional Cosines

- What are the directional cosines?

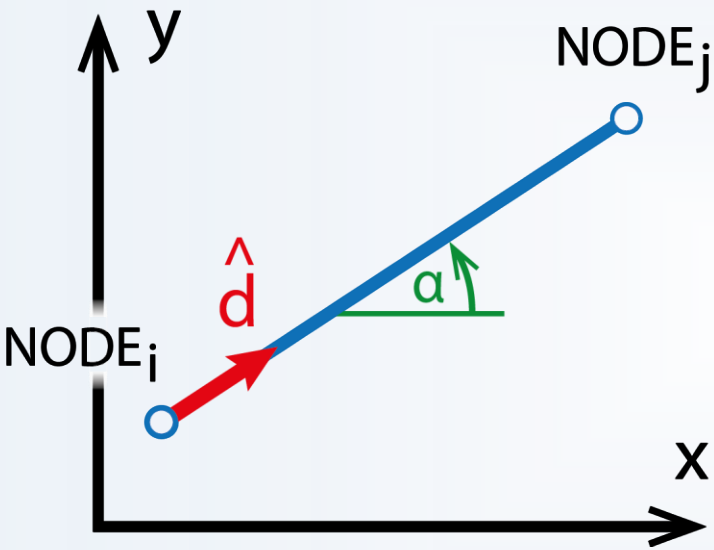


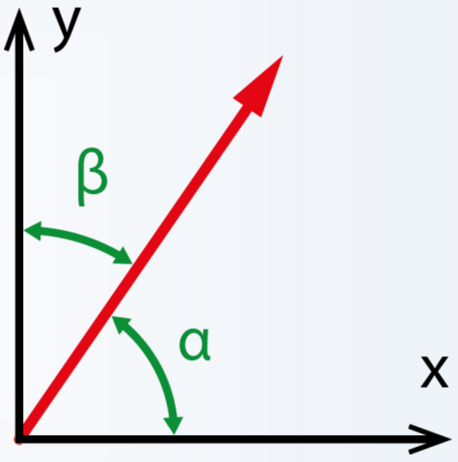


- Note that in 2D: $\sin\alpha = \cos\beta$
  $\cos^2\alpha + \cos^2\beta = 1$
- In 3D too: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Directional Cosines

- What are the directional cosines?



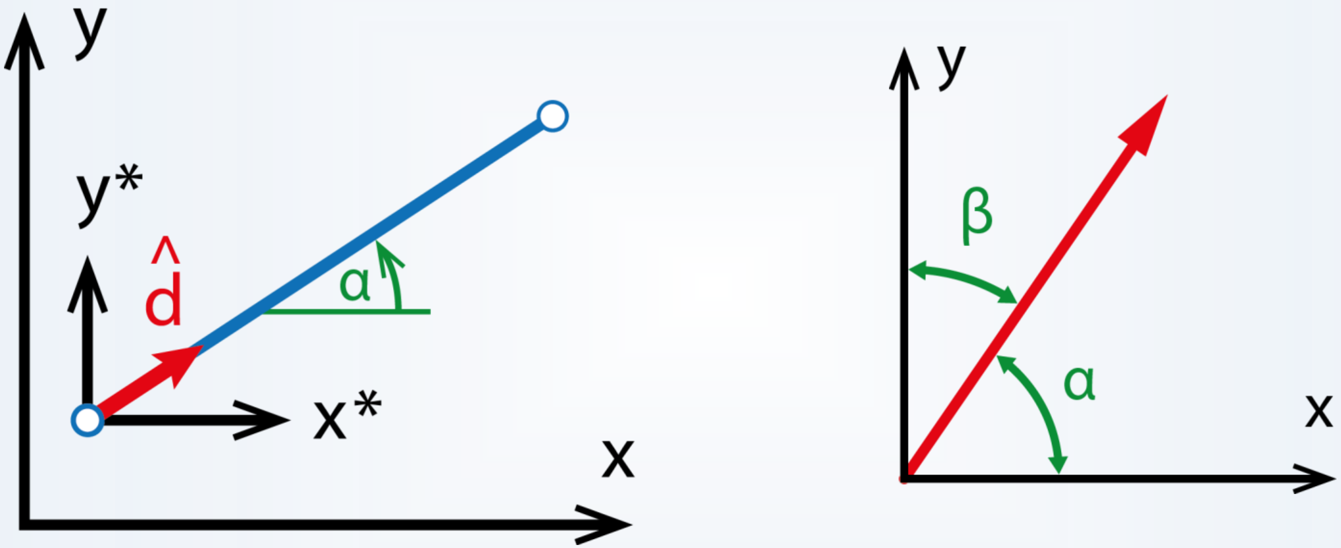


- For the case of our truss member:

$$\left.\begin{array}{c}\mathbf{d} = \mathbf{NODE_j} - \mathbf{NODE_i} \\ L = \|\mathbf{d}\|_2\end{array}\right\} \hat{\mathbf{d}} = \mathbf{d}/L$$

Directional Cosines

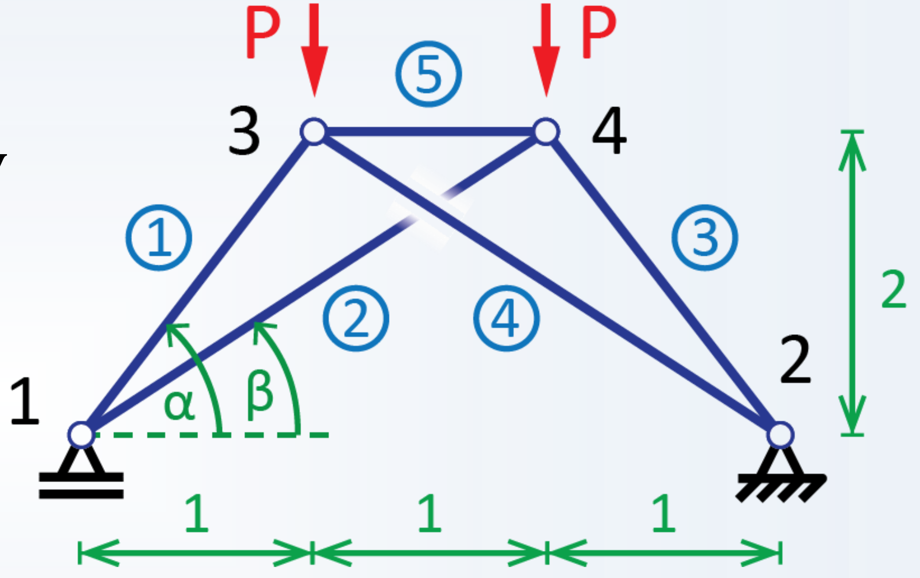
- What are the directional cosines?



- The directional cosines are used to construct $[\mathbf{B}^{\mathrm{T}}]$

Automatic Assembly of B^T

- Given
  - List of coordinates
  - List of element connectivity



$$\mathbf{NODE} = \begin{bmatrix} 0 & 0 \\ 3 & 0 \\ 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\mathbf{ELEM} = \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 2 & 4 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$$

$$\mathbf{d}_1 = \mathbf{NODE}_3 - \mathbf{NODE}_1 = [1 \quad 2] - [0 \quad 0] = [1 \quad 2]$$
$$\mathbf{d}_2 = \mathbf{NODE}_4 - \mathbf{NODE}_1 = [2 \quad 2] - [0 \quad 0] = [2 \quad 2]$$
$$\mathbf{d}_3 = \mathbf{NODE}_4 - \mathbf{NODE}_2 = [2 \quad 2] - [3 \quad 0] = [-1 \quad 2]$$
$$\mathbf{d}_4 = \mathbf{NODE}_3 - \mathbf{NODE}_2 = [1 \quad 2] - [3 \quad 0] = [-2 \quad 2]$$
$$\mathbf{d}_5 = \mathbf{NODE}_4 - \mathbf{NODE}_3 = [2 \quad 2] - [1 \quad 2] = [1 \quad 0]$$

Automatic Assembly of B^T

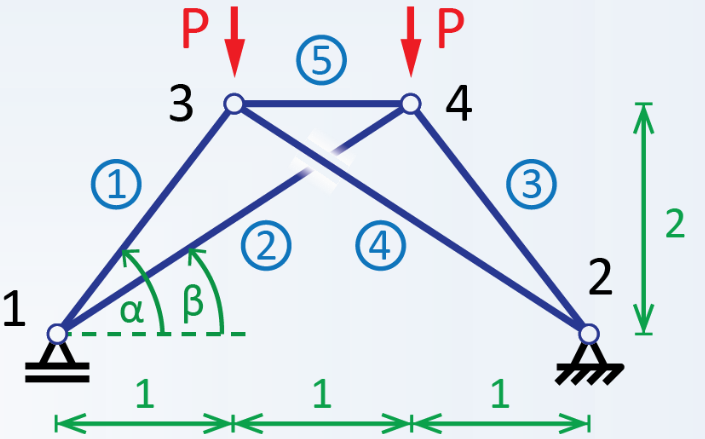
$$\hat{\mathbf{d}}_1 = \mathbf{d}_1/L_1 = [\cos\alpha_1 \quad \sin\alpha_1]$$

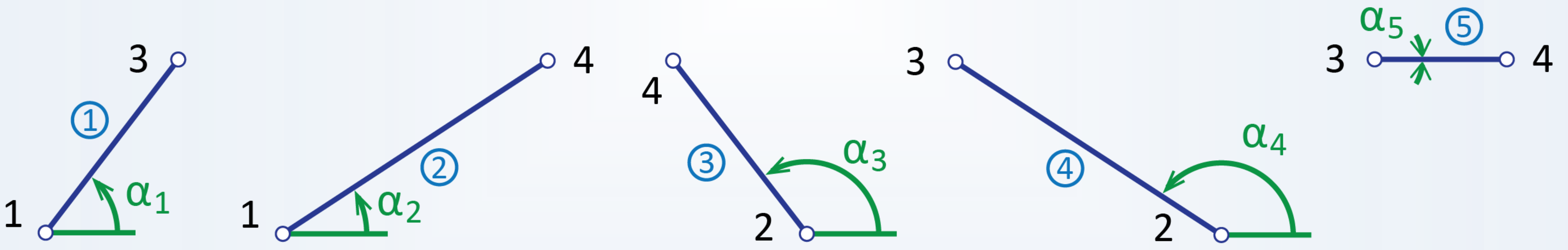
$$\hat{\mathbf{d}}_2 = \mathbf{d}_2/L_2 = [\cos\alpha_2 \quad \sin\alpha_2]$$

$$\hat{\mathbf{d}}_3 = \mathbf{d}_3/L_3 = [\cos\alpha_3 \quad \sin\alpha_3]$$

$$\hat{\mathbf{d}}_4 = \mathbf{d}_4/L_4 = [\cos\alpha_4 \quad \sin\alpha_4]$$

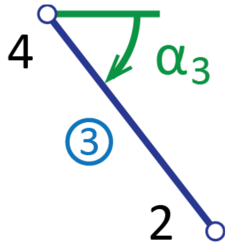
$$\hat{\mathbf{d}}_5 = \mathbf{d}_5/L_5 = [\cos\alpha_5 \quad \sin\alpha_5]$$





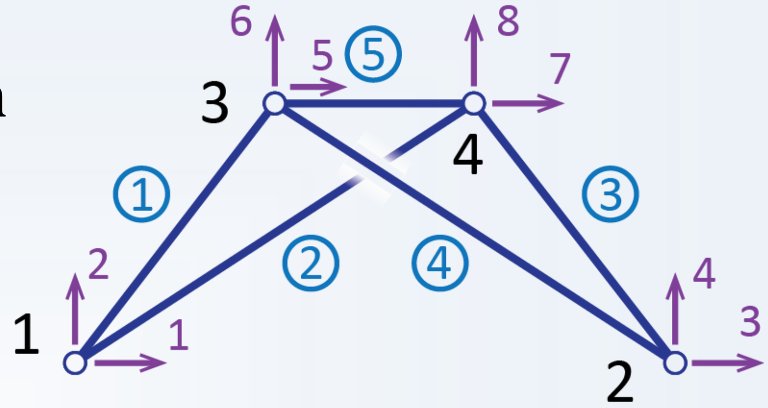
- Why is α_3 not this angle instead?

  A: Element is defined from (2) to (4), not from (4) to (2).



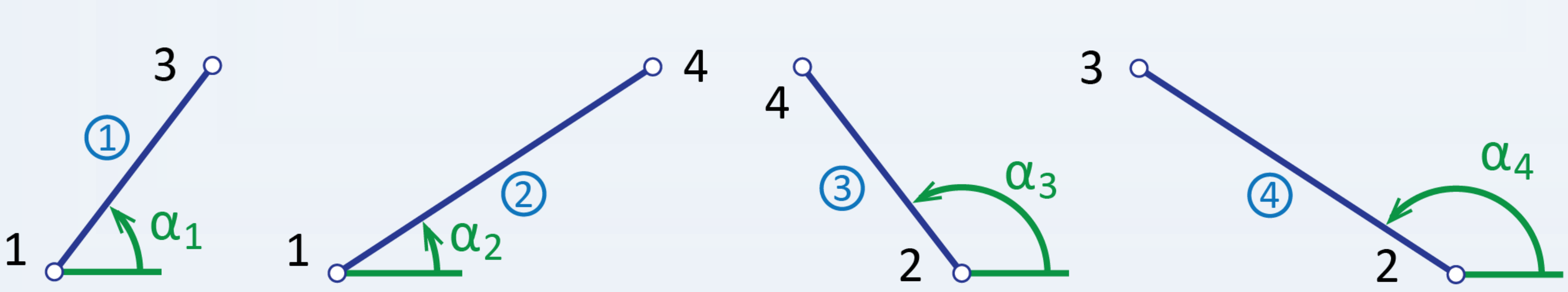
Automatic Assembly of B^{T}

- Looking at the degrees-of-freedom of the structure



$$\begin{bmatrix} -\hat{\mathbf{d}}_{1x} & -\hat{\mathbf{d}}_{2x} & 0 & 0 & 0 \\ -\hat{\mathbf{d}}_{1y} & -\hat{\mathbf{d}}_{2y} & 0 & 0 & 0 \\ 0 & 0 & -\hat{\mathbf{d}}_{3x} & -\hat{\mathbf{d}}_{4x} & 0 \\ 0 & 0 & -\hat{\mathbf{d}}_{3y} & -\hat{\mathbf{d}}_{4y} & 0 \\ \hat{\mathbf{d}}_{1x} & 0 & 0 & \hat{\mathbf{d}}_{4x} & -\hat{\mathbf{d}}_{5x} \\ \hat{\mathbf{d}}_{1y} & 0 & 0 & \hat{\mathbf{d}}_{4y} & -\hat{\mathbf{d}}_{5y} \\ 0 & \hat{\mathbf{d}}_{2x} & \hat{\mathbf{d}}_{3x} & 0 & \hat{\mathbf{d}}_{5x} \\ 0 & \hat{\mathbf{d}}_{2y} & \hat{\mathbf{d}}_{3y} & 0 & \hat{\mathbf{d}}_{5y} \end{bmatrix} = \begin{bmatrix} \mathbf{B} \\ \boxed{\mathbf{B}_{\mathbf{nr}}} \end{bmatrix}^{\mathbf{T}}$$

DOFs with supports (reactions)



Ground Structure Formulation

- The least-volume structure subjected to stress constraints is

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & \mathbf{a}^{\mathrm{T}}\mathbf{l} \\
\text{s.t.} \quad & \mathbf{Ku} = \mathbf{f} \\
& -\sigma_c \leq \sigma_i \leq \sigma_t \quad \text{if } a_i > 0 \\
& a_i \geq 0
\end{aligned}
$$

Elastic Formulation

- Remarks:
  - Only sizing the members. Nodal locations and connectivity are fixed.
  - Stress constraint does not apply for a vanished member.
  - Stiffness matrix includes: force eq, compatibility and stress-strain relations.

Ground Structure Formulation

- Enforcing force equilibrium only:
(no compatibility or stress-strain)

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^{\mathrm{T}}\mathbf{l} \\ \text{s.t.} \quad & \mathbf{B}^{\mathrm{T}}\mathbf{n} = \mathbf{f} \\ & -\sigma_c \leq \sigma_i \leq \sigma_t \quad \text{if } a_i > 0 \\ & a_i \geq 0 \end{aligned}$$

Plastic Formulation

- Remarks:
  - Only sizing the members. Nodal locations and connectivity are fixed.
  - Stress constraint does not apply for a vanished member.
  - For statically indeterminate structures [**B**] is not symmetric or invertible.

Ground Structure Formulation

Q: We are only sizing members. How do we expect to do topology optimization?

A: The ground structure method relies on raw power: highly interconnected truss with many nodes and discards the less useful ones. Note that the starting structure will not be determinate.

Q: Can a member have stress or strain if its area is zero?

A: In theory yes. Given displacements **u**, the strain in a member is $\varepsilon = \Delta u/l$ and the stress is $\sigma = E\varepsilon$. No area involved!

Q: In the plastic formulation: Is the optimal structure valid? (compatibility and $\sigma - \varepsilon$)

A: The size of [**B**] is $N_{dof} \times N_e$. We know that the optimal solution will have at most N_{dof} non-zero basic variables → the reduced $[B]$ is square and the structure is statically determinate: It automatically complies with compatibility and $\sigma - \varepsilon$.

Slack Variables & LP Form

- Plastic formulation

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^{\mathrm{T}}\mathbf{l} \\ \text{s.t.} \quad & \mathbf{B}^{\mathrm{T}}\mathbf{n} = \mathbf{f} \\ & -\sigma_c \leq \sigma_i \leq \sigma_t \quad \text{if } a_i > 0 \\ & a_i \geq 0 \end{aligned}$$

- Discontinuity in the stress constraint (vanishing)
  - Rewrite in terms of member force (multiply by **a**)

$$-\sigma_c a_i \leq n_i \leq \sigma_t a_i \quad \forall\, a_i \geq 0$$

Slack Variables & LP Form

- Plastic formulation

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^{\mathrm{T}}\mathbf{l} \\ \text{s.t.} \quad & \mathbf{B}^{\mathrm{T}}\mathbf{n} = \mathbf{f} \\ & -\sigma_c a_i \leq n_i \leq \sigma_t a_i \quad \forall\, a_i \geq 0 \end{aligned}$$

- Introducing slack variables in the inequalities
  - The positive constants multiplying the slack variables simplify the resulting expressions

$$\left.\begin{aligned} n_i + 2\frac{\sigma_0}{\sigma_c} s_i^- &= \sigma_t a_i \\ -n_i + 2\frac{\sigma_0}{\sigma_t} s_i^+ &= \sigma_c a_i \end{aligned}\right\} \begin{aligned} a_i &= \frac{s_i^+}{\sigma_t} + \frac{s_i^-}{\sigma_c} \\ n_i &= s_i^+ - s_i^- \end{aligned}$$

, with $\sigma_0 = (\sigma_t + \sigma_c)/2$

Slack Variables & LP Form

- Plastic formulation (rewritten in terms of $\mathbf{s}^+$ and $\mathbf{s}^-$)

$$\begin{aligned} \min_{\mathbf{s}^+,\mathbf{s}^-} \quad & \left(\frac{\mathbf{s}^+}{\sigma_t} + \frac{\mathbf{s}^-}{\sigma_c}\right)^{\mathbf{T}} \mathbf{l} \\ \text{s. t.} \quad & \mathbf{B}^{\mathbf{T}}\left(\mathbf{s}^+ - \mathbf{s}^-\right) = \mathbf{f} \\ & \mathbf{s}^+, \mathbf{s}^- > \mathbf{0} \end{aligned}$$

- Reorganizing a little…

$$\begin{aligned} \min_{\mathbf{s}^+,\mathbf{s}^-} \quad & \{\mathbf{l}/\sigma_t \vdots \mathbf{l}/\sigma_c\} \begin{Bmatrix} \mathbf{s}_i^+ \\ \cdots \\ \mathbf{s}_i^- \end{Bmatrix} \\ \text{s. t.} \quad & [\mathbf{B}^{\mathbf{T}} \vdots -\mathbf{B}^{\mathbf{T}}] \begin{Bmatrix} \mathbf{s}_i^+ \\ \cdots \\ \mathbf{s}_i^- \end{Bmatrix} = \mathbf{f} \\ & \mathbf{s}^+, \mathbf{s}^- > \mathbf{0} \end{aligned}$$

Slack Variables & LP Form

$$\min_{\mathbf{s}^+,\mathbf{s}^-} \quad \{\mathbf{1}/\sigma_t : \mathbf{1}/\sigma_c\} \begin{Bmatrix} \mathbf{s}_i^+ \\ \cdots \\ \mathbf{s}_i^- \end{Bmatrix}$$

$$\text{s.t.} \quad [\mathbf{B}^{\mathrm{T}} : -\mathbf{B}^{\mathrm{T}}] \begin{Bmatrix} \mathbf{s}_i^+ \\ \cdots \\ \mathbf{s}_i^- \end{Bmatrix} = \mathbf{f}$$

$$\mathbf{s}^+, \mathbf{s}^- > \mathbf{0}$$

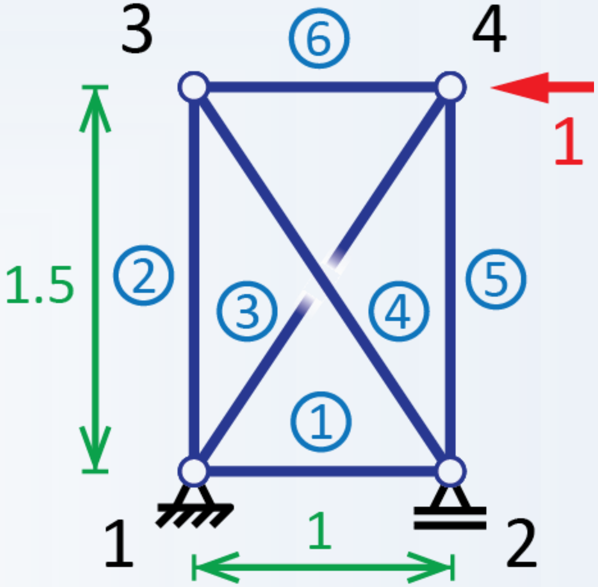
- What is so special about this?
  - The variables have been doubled → **bad**
  - Reduced the number of constraints → **good**
  - Linear Programming form → **AWESOME!**
    - The problem can be solved fast (interior-point method)
    - Optimum is global

Basic Example

- Given the following problem

$$\mathbf{NODE} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1.5 \\ 1 & 1.5 \end{bmatrix}$$

$$\mathbf{ELEM} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 3 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$



Assume that

$$\sigma_t = \sigma_c = 1$$

- Structure is indeterminate (redundant)
  - No redundancy → no alternative topologies to chose from

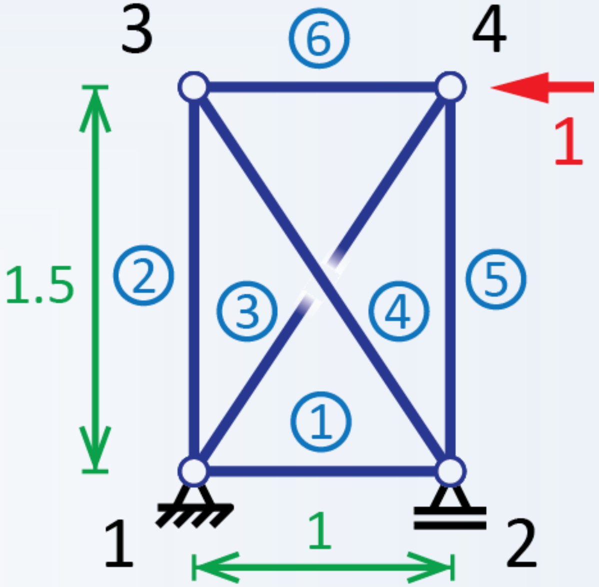
Basic Example

- The LP variables are:

$$\mathbf{S} = \begin{Bmatrix} s_1^+ \\ s_2^+ \\ s_3^+ \\ s_4^+ \\ s_5^+ \\ s_6^+ \\ s_1^- \\ s_2^- \\ s_3^- \\ s_4^- \\ s_5^- \\ s_6^- \end{Bmatrix} = \begin{Bmatrix} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \\ S7 \\ S8 \\ S9 \\ S10 \\ S11 \\ S12 \end{Bmatrix}$$

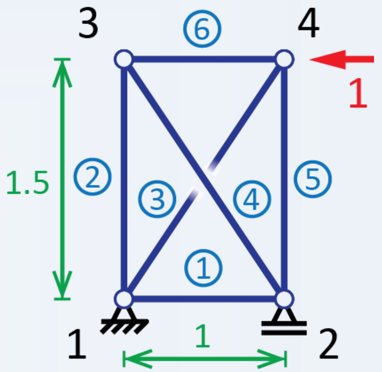
Associated with tension (S1–S6)

Associated with compression (S7–S12)



Basic Example

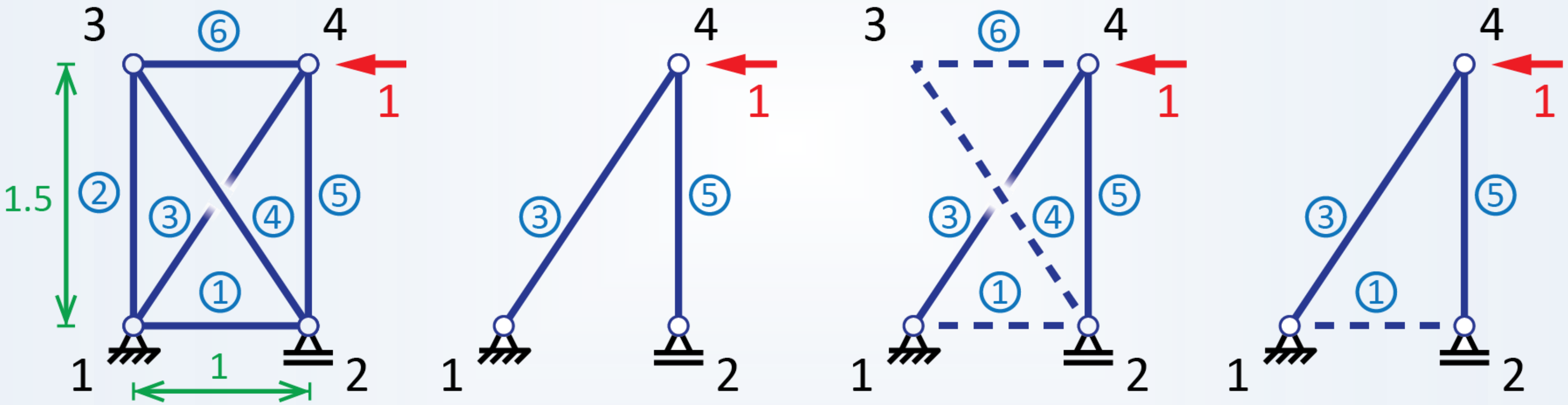
- Simplex Tableau
  - Basic members are (1), (3), (4), (5) and (6)



"linopt"	"restr"	slk$_1$	slk$_2$	slk$_3$	slk$_4$	slk$_5$	slk$_6$	slk$_7$	slk$_8$	slk$_9$	slk$_{10}$	S4	S1	S7	S10	S6	S12	S2	S8	S3	S9	S5	S11
"obj"	-5.5	0	1.0	0	4.5	0	0.167	0	5.5	1.5	0	0	2.0	0	3.61	2.0	0	1.67	1.33	3.61	0	0	3.0
slk$_1$	0.0	1	1.0	0	0.0	0	0.0	0	0.0	0.0	0	0	0.0	0	0.0	0.0	0	0.0	0.0	0.0	0	0	0.0
S4	0.0	0	0.0	0	0.0	0	1.2	0	0.0	0.0	0	1	0.0	0	-1.0	0.0	0	1.2	-1.2	0.0	0	0	0.0
slk$_3$	0.0	0	0.0	1	1.0	0	0.0	0	0.0	0.0	0	0	0.0	0	0.0	0.0	0	0.0	0.0	0.0	0	0	0.0
S12	0.0	0	0.0	0	1.0	0	0.667	0	0.0	0.0	0	0	0.0	0	0.0	-1.0	1	0.667	-0.667	0.0	0	0	0.0
slk$_5$	0.0	0	0.0	0	0.0	1	1.0	0	0.0	0.0	0	0	0.0	0	0.0	0.0	0	0.0	0.0	0.0	0	0	0.0
S7	0.0	0	-1.0	0	0.0	0	0.667	0	0.0	0.0	0	0	-1.0	1	0.0	0.0	0	0.667	-0.667	0.0	0	0	0.0
slk$_7$	0.0	0	0.0	0	0.0	0	0.0	1	1.0	0.0	0	0	0.0	0	0.0	0.0	0	0.0	0.0	0.0	0	0	0.0
S5	1.5	0	0.0	0	-1.5	0	-1.0	0	-1.5	-1.0	0	0	0.0	0	0.0	0.0	0	-1.0	1.0	0.0	0	1	-1.0
S9	1.8	0	0.0	0	-1.8	0	-1.2	0	-1.8	0.0	0	0	0.0	0	0.0	0.0	0	-1.2	1.2	-1.0	1	0	0.0
slk$_{10}$	0.0	0	0.0	0	0.0	0	0.0	0	0.0	1.0	1	0	0.0	0	0.0	0.0	0	0.0	0.0	0.0	0	0	0.0

Basic Example

- Post-processing of optimal structure



Initial structure

Optimal structure

Basic Members

Reduced Optimal Structure (ROS)

- Structure will be in equilibrium, but may be unstable.
    - Some basic members may require a minimum area to stabilize the structure

Fast Ground Structure Generation

- Easy to get carried away and connect every node with every other node
  - In general **we do not want overlapping bars**
  - Example: Assume $P = 1$ and $\sigma_t = 1$

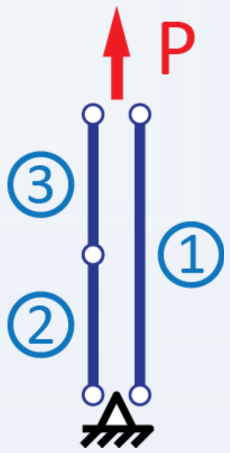
$$a_1 = 1.0$$

$$a_1 = 1.0 \quad a_2 = a_3 = 0.0$$

$$a_1 = 0.0 \quad a_2 = a_3 = 1.0$$

$$a_1 = 0.5 \quad a_2 = a_3 = 0.5$$

Solution is not unique



Fast Ground Structure Generation

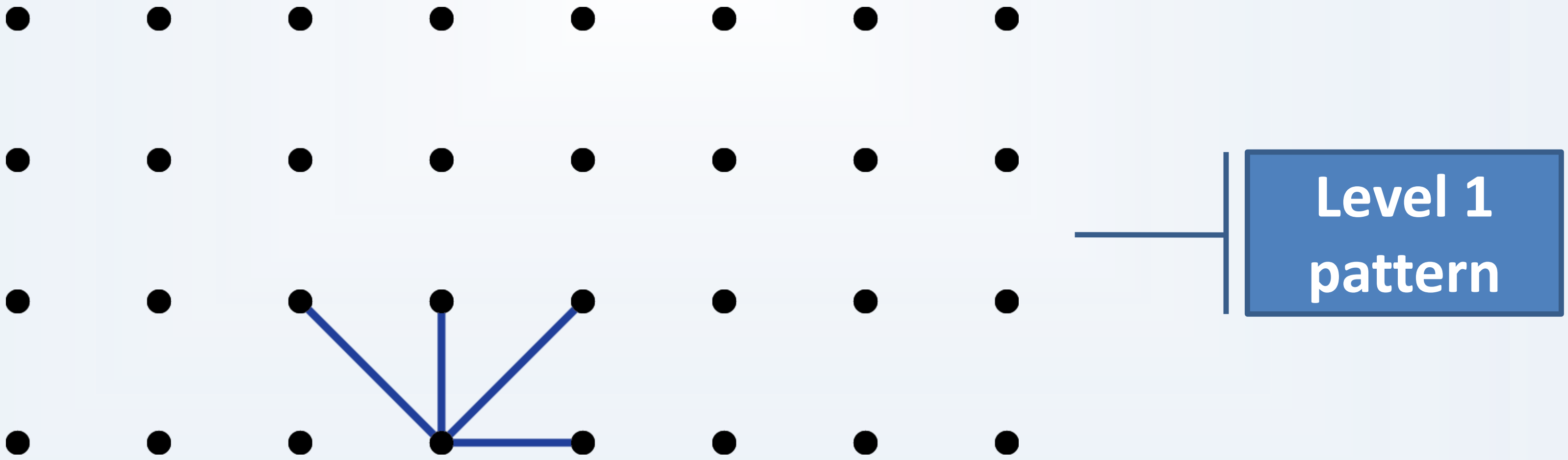
- The idea is to "stamp" a pattern in all nodes of a grid
  - This pattern has no overlapping bars
  - This only works for structured—orthogonal grids

Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

- Pattern is created with a user-defined level
  - Structure is more redundant with higher levels

- Looking at the pattern for a single node



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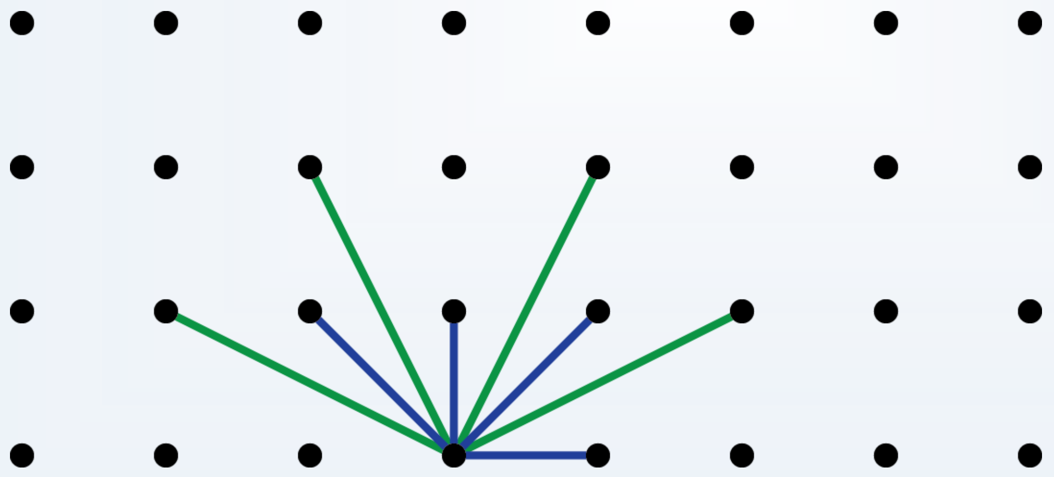

Level 2 pattern

Sokol, T “A 99 line code for discretized Michell truss optimization written in Mathematica” Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

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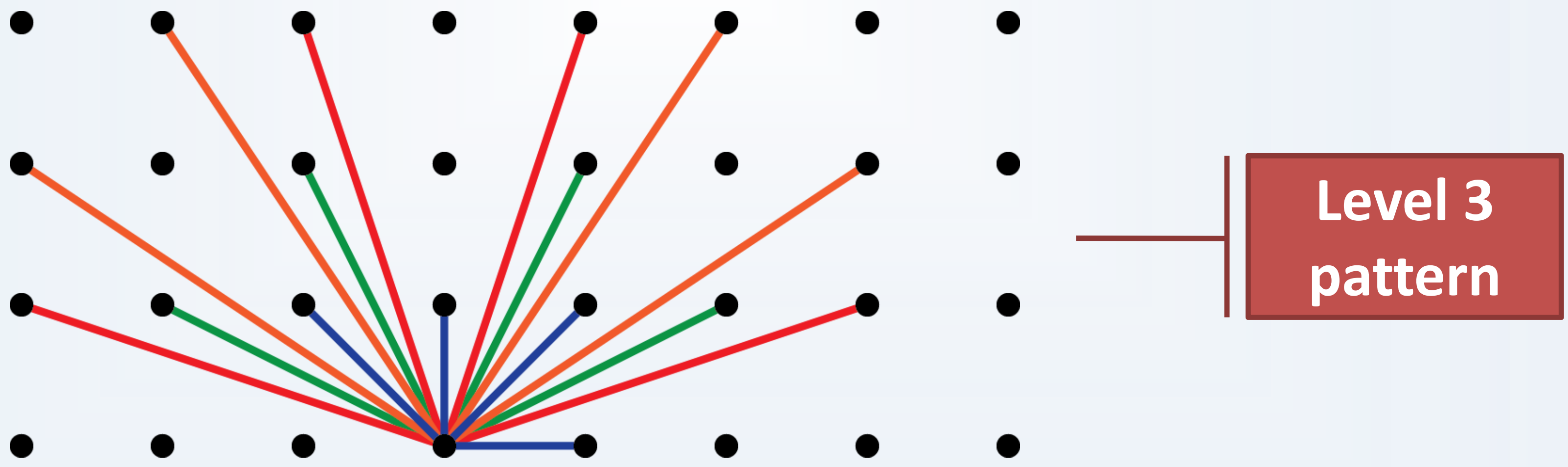
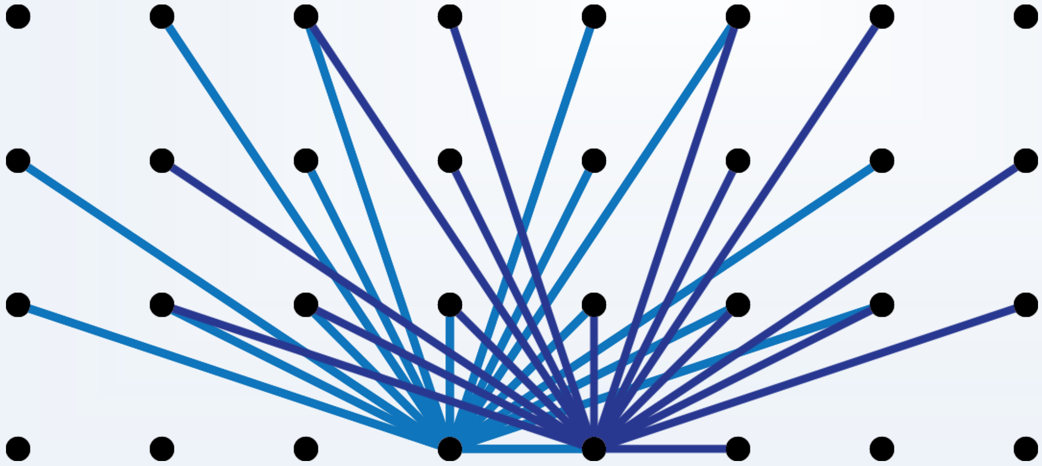


Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

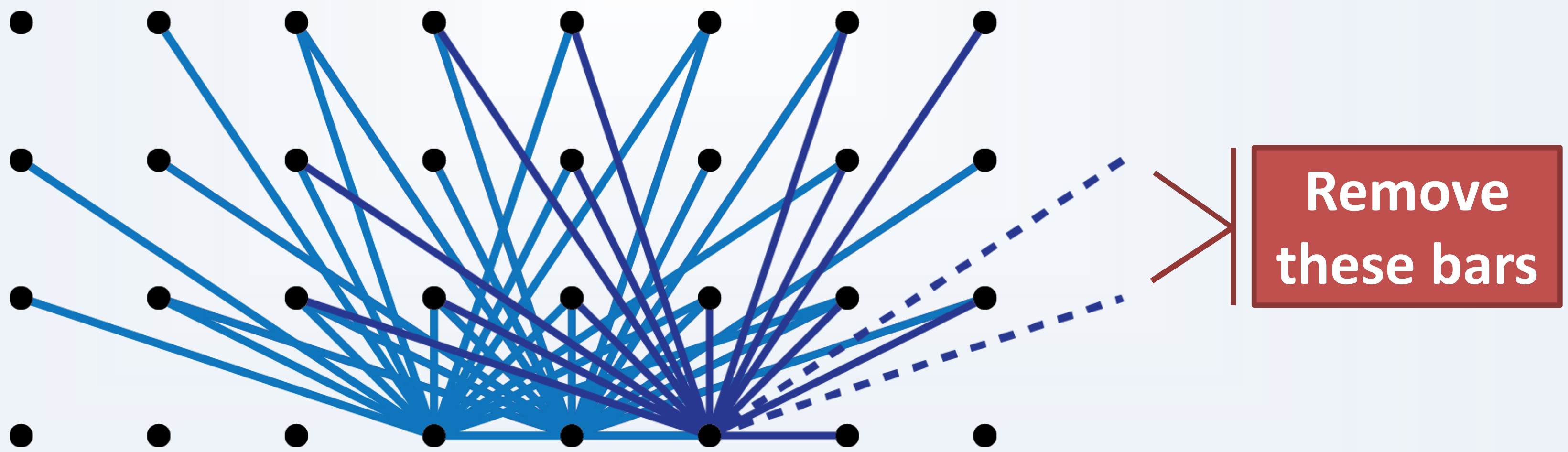
- Stamping the pattern in other nodes…



Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

- Stamping the pattern in other nodes…
- Keep an eye for members exiting the grid…

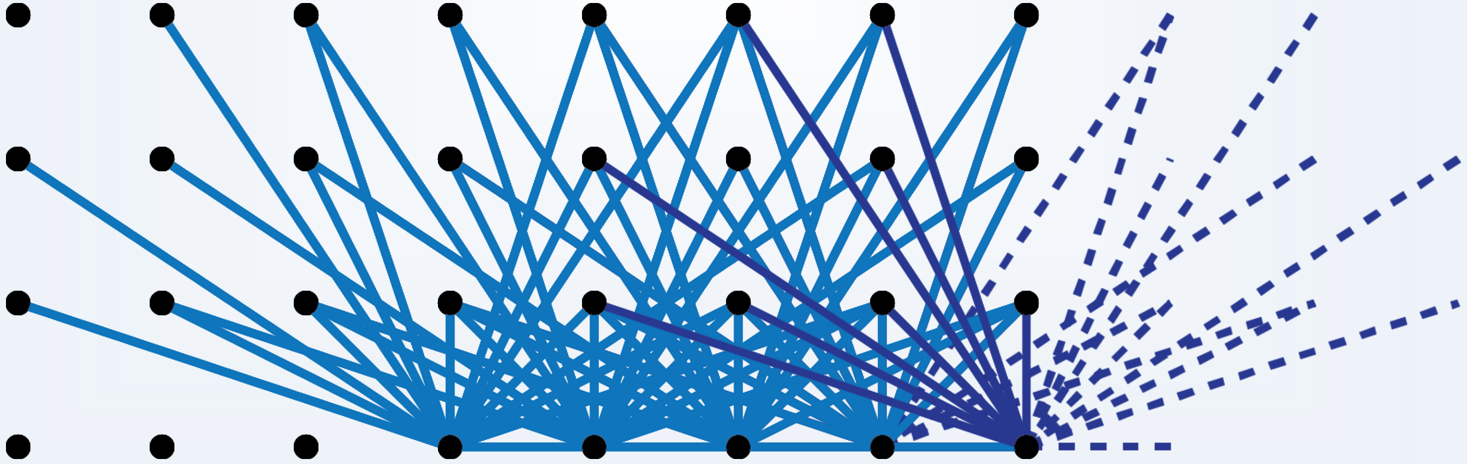


Remove these bars

Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

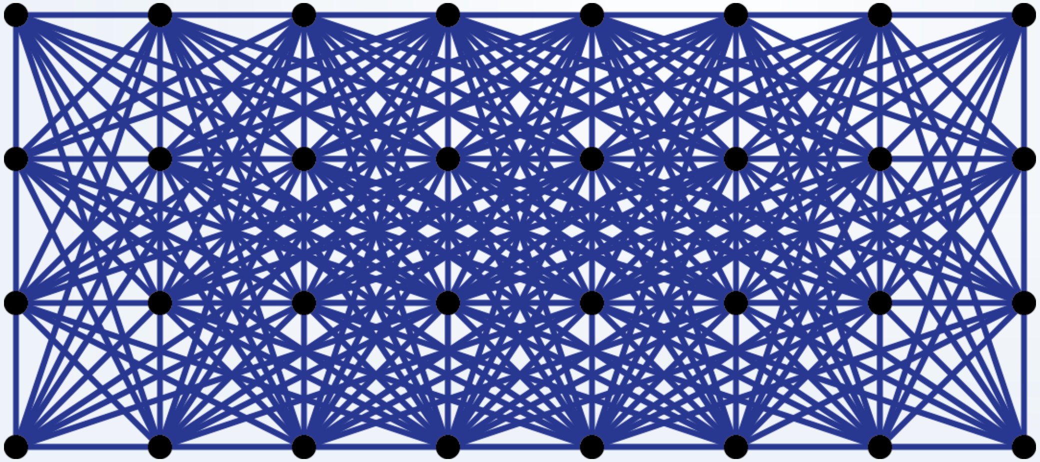
- Stamping the pattern in other nodes…
- Keep an eye for members exiting the grid…



Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Fast Ground Structure Generation

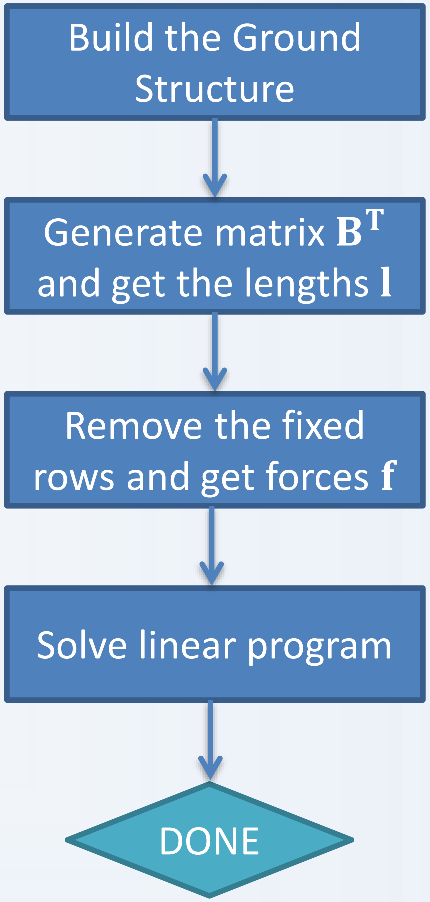
- Stamping the pattern in other nodes…
- Keep an eye for members exiting the grid…
- Repeat for all nodes in the grid…



Sokol, T "A 99 line code for discretized Michell truss optimization written in Mathematica" Structural and multidisciplinary optimization 43(2), pp. 181-190, (2010)

Summary

- Flowchart
  - No external loops
  - Iterations are done by the interior-point algorithm
  - Might require post-processing
  - Ground structure must be built with no overlapping members
  - Can be extended to domains other than a regular grid: Provided that you can construct a GS with no overlapping members



Build the Ground Structure

Generate matrix $\mathbf{B}^T$ and get the lengths $\mathbf{l}$

Remove the fixed rows and get forces $\mathbf{f}$

Solve linear program

DONE

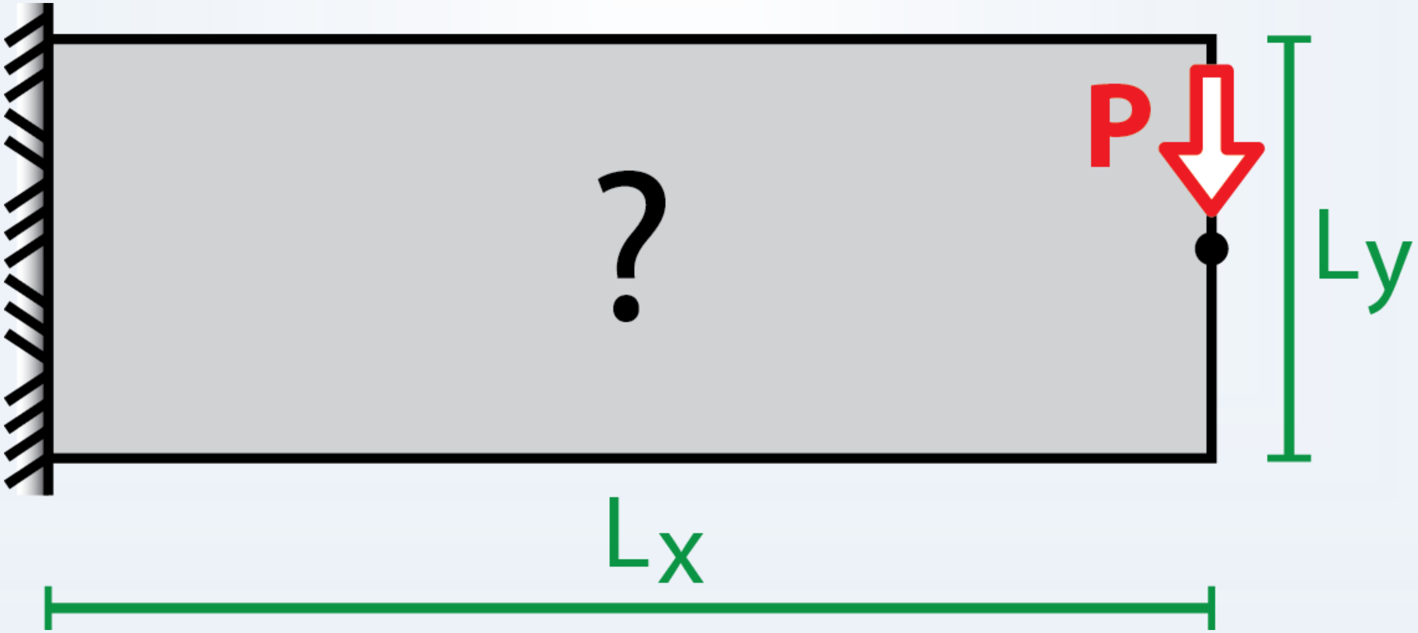
Large Example

- Cantilever domain

$L_x = 3 \quad L_y = 1 \quad P = 1$

30×10 mesh → 31×11 point grid

Level 10 connectivity → 19632 members



Large Example

- Cantilever domain

  $L_x = 3 \quad L_y = 1 \quad P = 1$

  30 × 10 mesh → 31 × 11 point grid

  Level 10 connectivity → 19,632 members

Large Example

- How does 19,632 members look like?

- This is not really a “large problem”:
  - This method can easily handle millions of members

Large Example

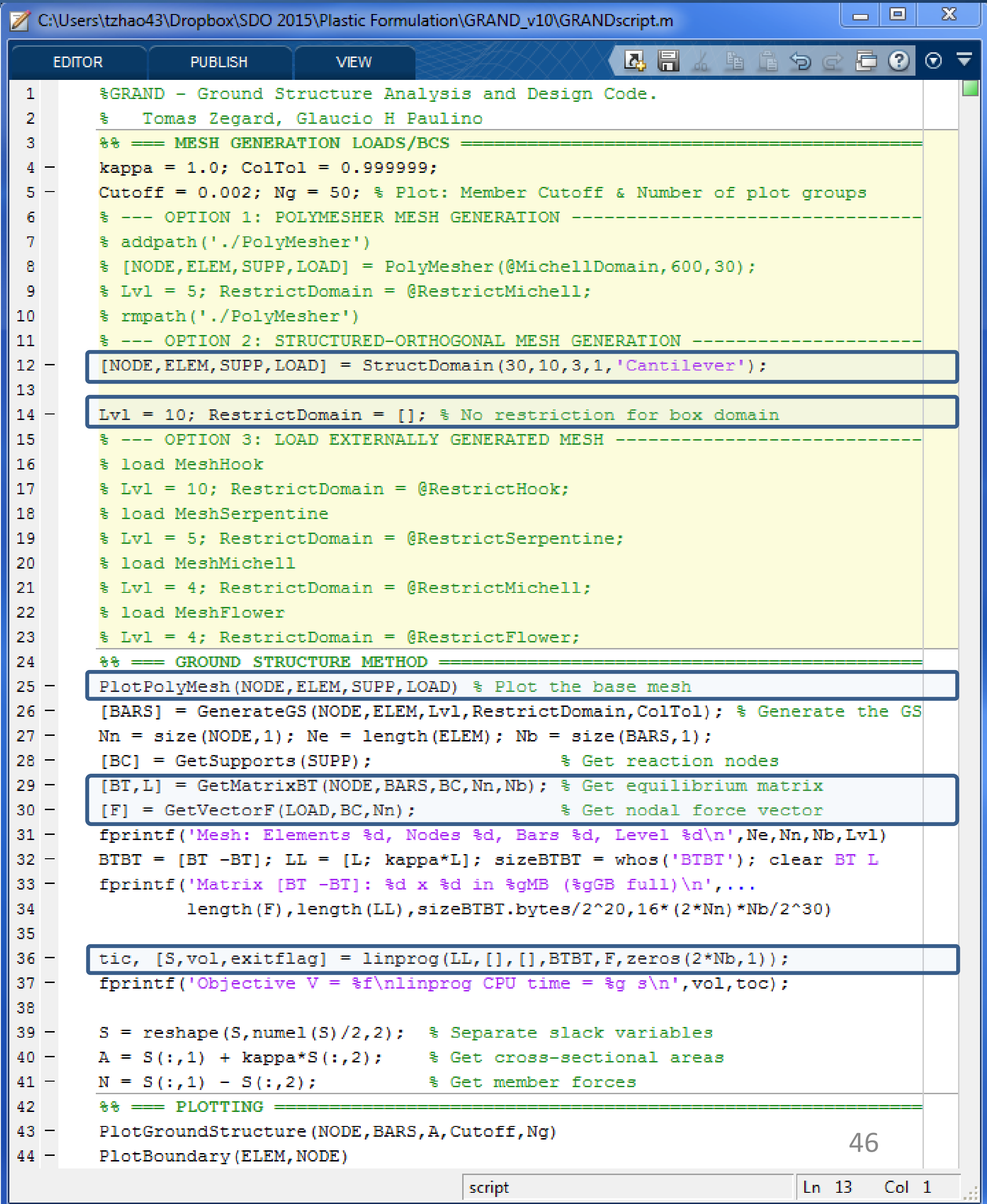
Define domain size and the number of grids

Define level of Ground Structure

Generate Ground Structure

Obtain equilibrium matrix and force vector

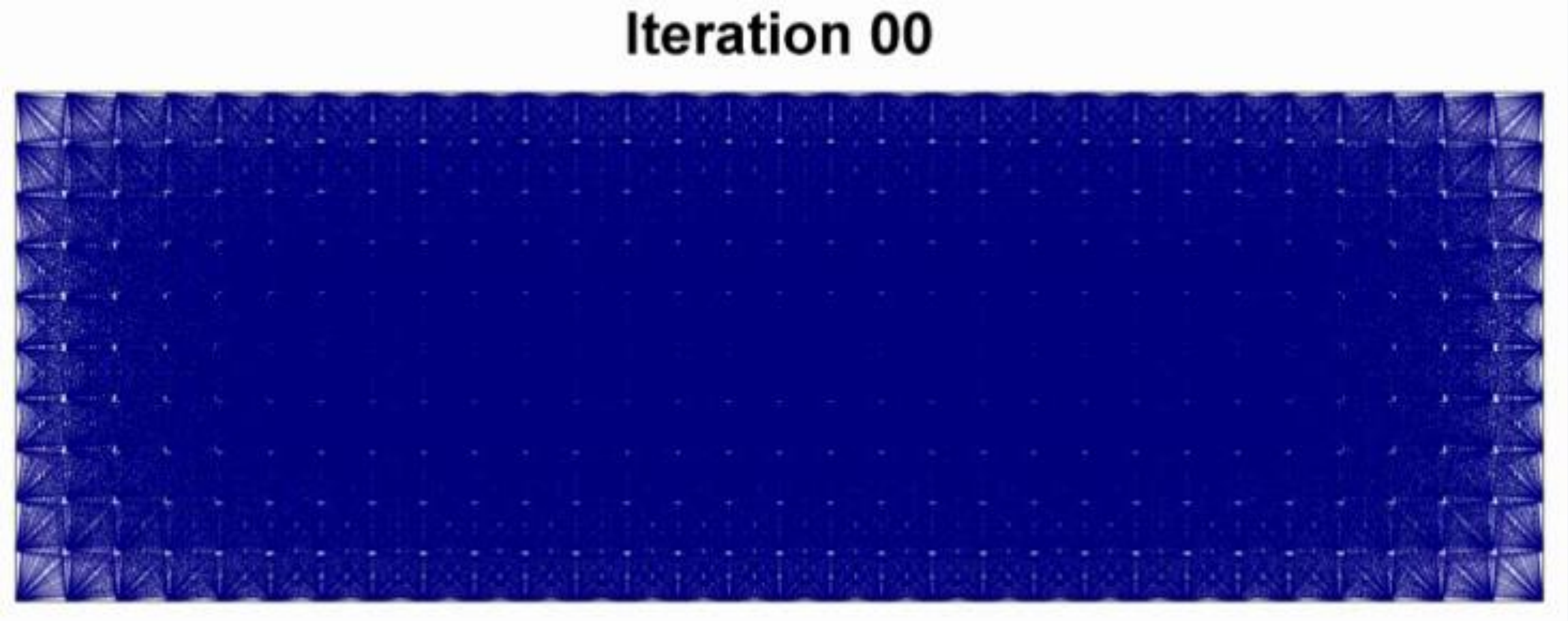
Call LP optimizer



```
%GRAND - Ground Structure Analysis and Design Code.
%   Tomas Zegard, Glaucio H Paulino
%% === MESH GENERATION LOADS/BCS ==========================================
kappa = 1.0; ColTol = 0.999999;
Cutoff = 0.002; Ng = 50; % Plot: Member Cutoff & Number of plot groups
% --- OPTION 1: POLYMESHER MESH GENERATION --------------------------------
% addpath('./PolyMesher')
% [NODE,ELEM,SUPP,LOAD] = PolyMesher(@MichellDomain,600,30);
% Lvl = 5; RestrictDomain = @RestrictMichell;
% rmpath('./PolyMesher')
% --- OPTION 2: STRUCTURED-ORTHOGONAL MESH GENERATION ---------------------
[NODE,ELEM,SUPP,LOAD] = StructDomain(30,10,3,1,'Cantilever');

Lvl = 10; RestrictDomain = []; % No restriction for box domain
% --- OPTION 3: LOAD EXTERNALLY GENERATED MESH ----------------------------
% load MeshHook
% Lvl = 10; RestrictDomain = @RestrictHook;
% load MeshSerpentine
% Lvl = 5; RestrictDomain = @RestrictSerpentine;
% load MeshMichell
% Lvl = 4; RestrictDomain = @RestrictMichell;
% load MeshFlower
% Lvl = 4; RestrictDomain = @RestrictFlower;
%% === GROUND STRUCTURE METHOD ============================================
PlotPolyMesh(NODE,ELEM,SUPP,LOAD) % Plot the base mesh
[BARS] = GenerateGS(NODE,ELEM,Lvl,RestrictDomain,ColTol); % Generate the GS
Nn = size(NODE,1); Ne = length(ELEM); Nb = size(BARS,1);
[BC] = GetSupports(SUPP);                 % Get reaction nodes
[BT,L] = GetMatrixBT(NODE,BARS,BC,Nn,Nb); % Get equilibrium matrix
[F] = GetVectorF(LOAD,BC,Nn);             % Get nodal force vector
fprintf('Mesh: Elements %d, Nodes %d, Bars %d, Level %d\n',Ne,Nn,Nb,Lvl)
BTBT = [BT -BT]; LL = [L; kappa*L]; sizeBTBT = whos('BTBT'); clear BT L
fprintf('Matrix [BT -BT]: %d x %d in %gMB (%gGB full)\n',...
        length(F),length(LL),sizeBTBT.bytes/2^20,16*(2*Nn)*Nb/2^30)

tic, [S,vol,exitflag] = linprog(LL,[],[],BTBT,F,zeros(2*Nb,1));
fprintf('Objective V = %f\nlinprog CPU time = %g s\n',vol,toc);

S = reshape(S,numel(S)/2,2);  % Separate slack variables
A = S(:,1) + kappa*S(:,2);    % Get cross-sectional areas
N = S(:,1) - S(:,2);          % Get member forces
%% === PLOTTING ===========================================================
PlotGroundStructure(NODE,BARS,A,Cutoff,Ng)
PlotBoundary(ELEM,NODE)
```

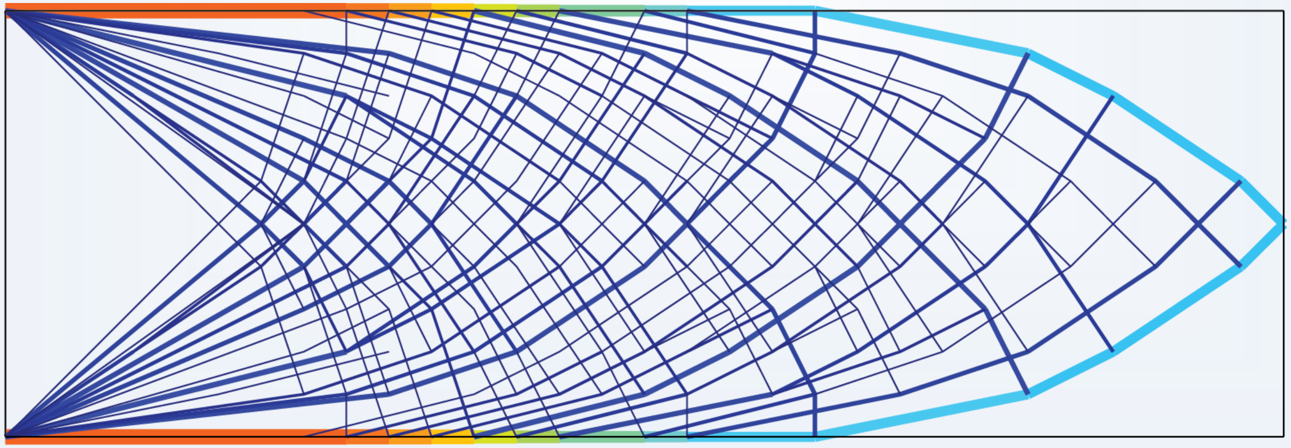
Large Example

- Video of the optimization
  - In reality iterations are done internally by the interior-point method → You will most likely never see this…



Large Example

- Final converged topology
  - Optimum is global
  - Takes ~5 seconds to run on an average computer



ELASTIC FORMULATION

Table of Contents

Simultaneous Formulation

- The least-compliance structure subjected to volume constraints is

$$
\begin{aligned}
\min_{\mathbf{a},\mathbf{u}} \quad & \mathbf{f}^{\mathrm{T}}\mathbf{u} \\
\text{s.t.} \quad & \mathbf{K}(\mathbf{a})\mathbf{u} = \mathbf{f} \\
& \sum_{j=1}^{nl} a_j L_j \leq V_{\max} \\
& a_j \geq 0
\end{aligned}
$$

Equilibrium constraint

nl : the number of bars

a_j : cross-sectional area of bar j

V_{max} : maximum allowed volume of the truss

$\mathbf{K}(\mathbf{a})$: global stiffness matrix

L_j : length of bar j

$\mathbf{f}$: global external force vector

Nested Formulation

- If the global stiffness matrix is nonsingular, we may eliminate the displacement vector from simultaneous formulation

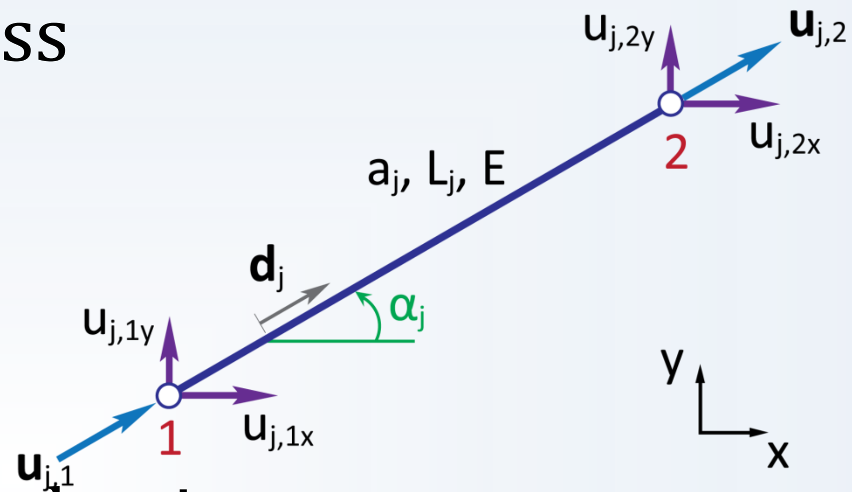
$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{f}^{\mathrm{T}}\mathbf{u}(\mathbf{a}) \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_j L_j \le V_{\max} \\ & a_j \ge 0 \\ \text{with} \quad & \mathbf{K}(\mathbf{a})\mathbf{u}(\mathbf{a}) = \mathbf{f} \end{aligned}$$

Equilibrium constraint

where $\mathbf{u}(\mathbf{a})$ is an implicit function defined through the equilibrium equations $\mathbf{K}(\mathbf{a})\mathbf{u}(\mathbf{a}) = \mathbf{f}$

Equilibrium Constraint: Geometry Equation

- A general bar *j* in a truss



$$\mathbf{d}_j = \begin{bmatrix} \cos \alpha_j \\ \sin \alpha_j \end{bmatrix}$$

- Displacement of the end points

$$\mathbf{u}_j = \begin{bmatrix} \mathbf{u}_{j,1} \\ \mathbf{u}_{j,2} \end{bmatrix}, \quad \text{where } \mathbf{u}_{j,1} = \begin{bmatrix} \mathrm{u}_{j,1x} \\ \mathrm{u}_{j,1y} \end{bmatrix} \text{ and } \mathbf{u}_{j,2} = \begin{bmatrix} \mathrm{u}_{j,2x} \\ \mathrm{u}_{j,2y} \end{bmatrix}$$

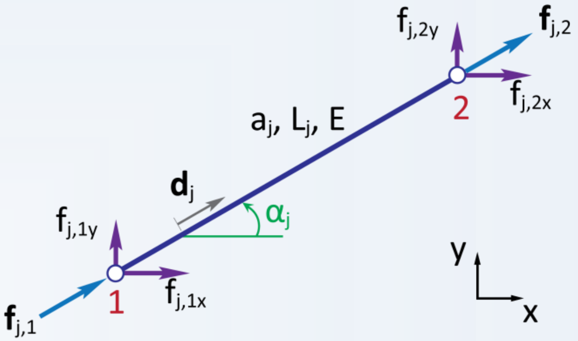
- Elongation δ_j

$$\delta_j = (\mathbf{u}_{j,2} - \mathbf{u}_{j,1}) \cdot \mathbf{d}_j = \mathbf{B}_j \mathbf{u}_j$$

Where

$$\mathbf{B}_j = \left[-\mathbf{d}_j{}^{\mathrm{T}} \ \mathbf{d}_j{}^{\mathrm{T}} \right] = [-\cos \alpha_j - \sin \alpha_j \ \cos \alpha_j \ \sin \alpha_j]$$

Equilibrium Constraint: Constitutive Equation



- External force $\boldsymbol{f}_j$ on the end points

$$\boldsymbol{f}_j = \mathbf{B}_j{}^{\mathrm{T}} n_j$$

- Force in the bar n_j

$$n_j = \sigma_j a_j = E\varepsilon_j a_j = E\delta_j a_j / L_j$$

- External force $\boldsymbol{f}_j$ can be rewritten as follow

$$\boldsymbol{f}_j = \frac{\mathbf{B}_j{}^{\mathrm{T}} E\delta_j a_j}{L_j} = \mathbf{B}_j{}^{\mathrm{T}} \left(\frac{E a_j}{L_j}\right) \mathbf{B}_j \mathbf{u}_j = \boldsymbol{K}_j \mathbf{u}_j$$

- Element stiffness matrix of bar j

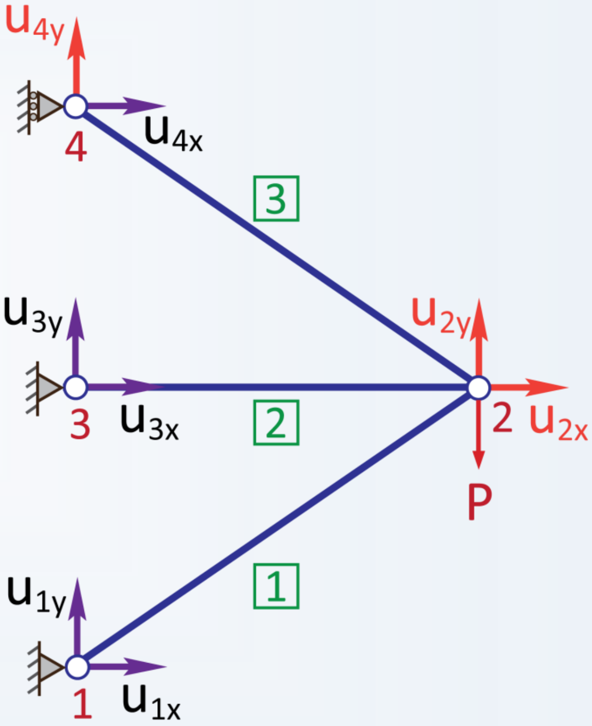
$$\boldsymbol{K}_j = \mathbf{B}_j{}^{\mathrm{T}} \left(\frac{E a_j}{L_j}\right) \mathbf{B}_j$$

$$\boldsymbol{K}_j(a_j) = a_j \mathbf{B}_j{}^{\mathrm{T}} \left(\frac{E}{L_j}\right) \mathbf{B}_j = a_j \boldsymbol{K}_j{}^0$$

Boolean Mapping Matrix

- Introduce a Boolean "mapping" matrix **C** which selects terms from global displacement **u** to element displacement $\mathbf{u}_j$

$$\mathbf{u}_j = \mathbf{C}_j\mathbf{u}$$



- Global displacement **u**

$$\mathbf{u} = \left[\mathrm{u}_{2x}\ \mathrm{u}_{2y}\ \mathrm{u}_{4y}\right]^{\mathrm{T}}$$

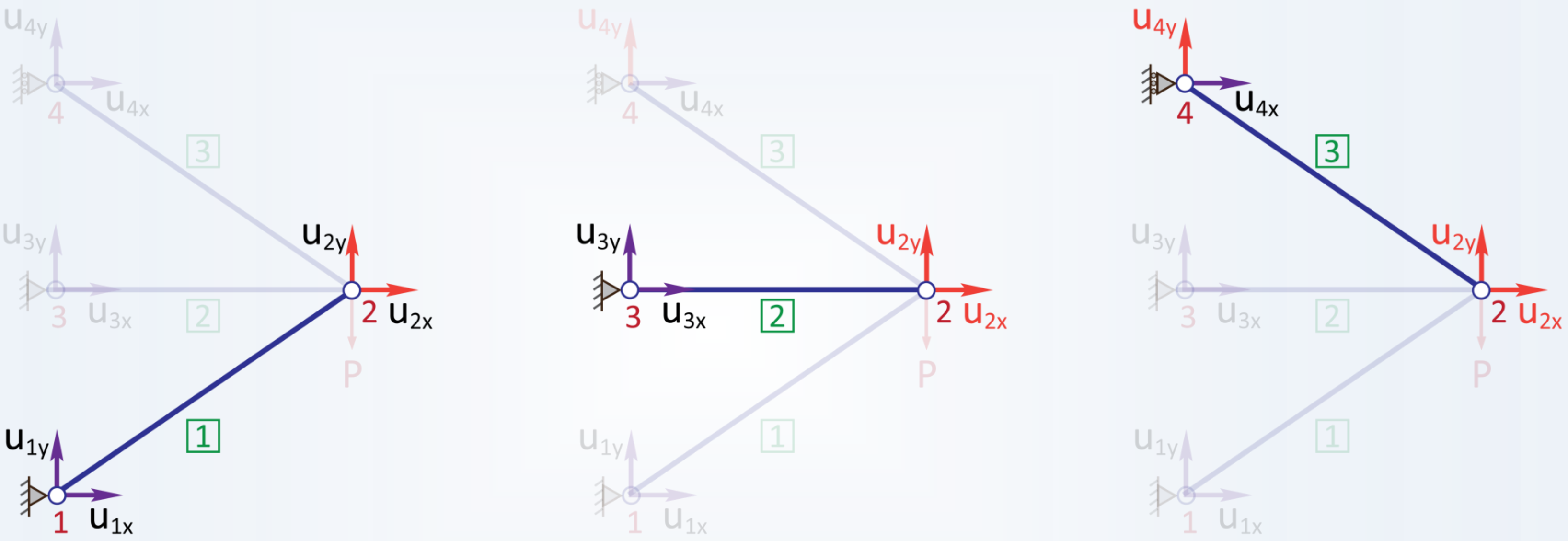
- Element displacement $\mathbf{u}_j$

$$\mathbf{u}_1 = [0\ 0\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}]^{\mathrm{T}},\ \mathbf{u}_2 = [0\ 0\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}]^{\mathrm{T}}$$

$$\mathbf{u}_3 = [0\ \mathrm{u}_{4y}\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}]^{\mathrm{T}}$$

Boolean Mapping Matrix

- Boolean Matrix **C**



$$\underbrace{\begin{bmatrix}0\\0\\u_{2x}\\u_{2y}\end{bmatrix}}_{\mathbf{u}_1} = \underbrace{\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\\0&1&0\end{bmatrix}}_{\mathbf{C}_1}\underbrace{\begin{bmatrix}u_{2x}\\u_{2y}\\u_{4y}\end{bmatrix}}_{\mathbf{u}}$$

$$\begin{bmatrix}0\\0\\u_{2x}\\u_{2y}\end{bmatrix} = \underbrace{\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\\0&1&0\end{bmatrix}}_{\mathbf{C}_2}\begin{bmatrix}u_{2x}\\u_{2y}\\u_{4y}\end{bmatrix}$$

$$\begin{bmatrix}0\\u_{4y}\\u_{2x}\\u_{2y}\end{bmatrix} = \underbrace{\begin{bmatrix}0&0&0\\0&0&1\\1&0&0\\0&1&0\end{bmatrix}}_{\mathbf{C}_3}\begin{bmatrix}u_{2x}\\u_{2y}\\u_{4y}\end{bmatrix}$$

Equilibrium Constraint: Equilibrium Condition

- Equilibrium equation of element *j* revisited

$$\boldsymbol{f}_j = a_j \boldsymbol{K}_j{}^0 \mathbf{u}_j = a_j \boldsymbol{K}_j{}^0 \mathbf{C}_j \mathbf{u}$$

$$\mathbf{C}_j{}^{\mathrm{T}} \boldsymbol{f}_j = \mathbf{C}_j{}^{\mathrm{T}} a_j \boldsymbol{K}_j{}^0 \mathbf{C}_j \mathbf{u}$$

- Constant matrix $\boldsymbol{k}_j{}^0$

$$\boldsymbol{K}_j{}^0 = \left(\frac{E}{L_j}\right) \mathbf{B}_j{}^{\mathrm{T}} \mathbf{B}_j = \left(\frac{E}{L_j}\right) \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

Note: $c = \cos \alpha_j$, $s = \sin \alpha_j$

- Global version of stiffness matrix and force vector of element *j*

$$\boldsymbol{K}_{g,j} = \mathbf{C}_j{}^{\mathrm{T}} a_j \boldsymbol{K}_j{}^0 \mathbf{C}_j \,, \qquad \mathbf{f}_{g,j} = \mathbf{C}_j{}^{\mathrm{T}} \boldsymbol{f}_j$$

Assembly of Global Matrix and Vector

- Global stiffness matrix and applied force vector

$$\mathbf{K} = \sum_{j=1}^{nl} \boldsymbol{K}_{g,j} = \sum_{j=1}^{nl} \mathbf{C}_j{}^{\mathrm{T}} a_j \boldsymbol{K}_j{}^{0} \mathbf{C}_j \qquad \mathbf{f} = \sum_{j=1}^{nl} \mathbf{f}_{g,j} = \sum_{j=1}^{nl} \boldsymbol{C}_{\boldsymbol{j}}{}^{\boldsymbol{T}} \boldsymbol{f}_{\boldsymbol{j}}$$

- For entire truss
  - Elongations

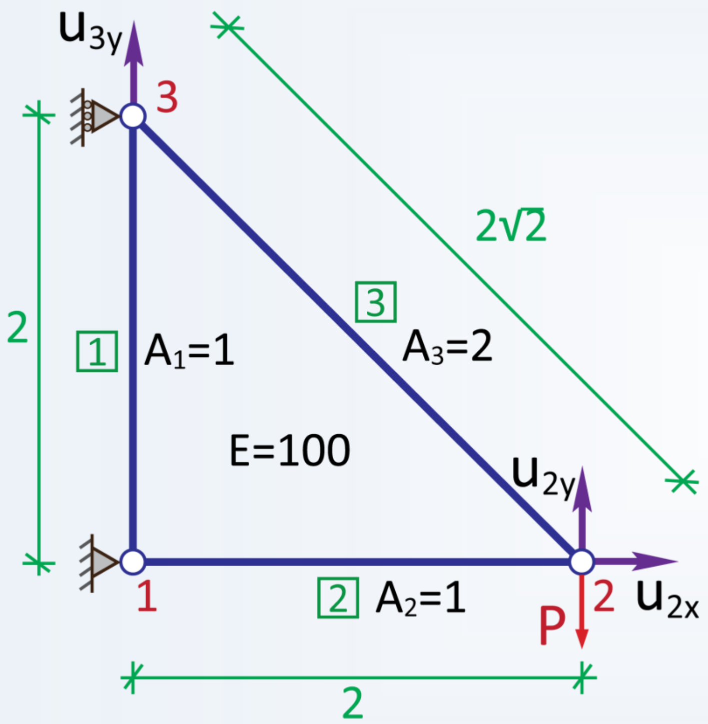
$$\boldsymbol{\delta} = \bar{\mathbf{B}}\mathbf{u} \qquad \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_1\mathbf{C}_1 \\ \vdots \\ \mathbf{B}_n\mathbf{C}_{\boldsymbol{n}} \end{bmatrix}$$

  - Stresses

$$\boldsymbol{\sigma} = \begin{bmatrix} 1/a_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/a_n \end{bmatrix} \bar{\mathbf{B}}^{-\mathrm{T}}\mathbf{f}$$

Example

- Three-bar truss



- Displacements of the bars

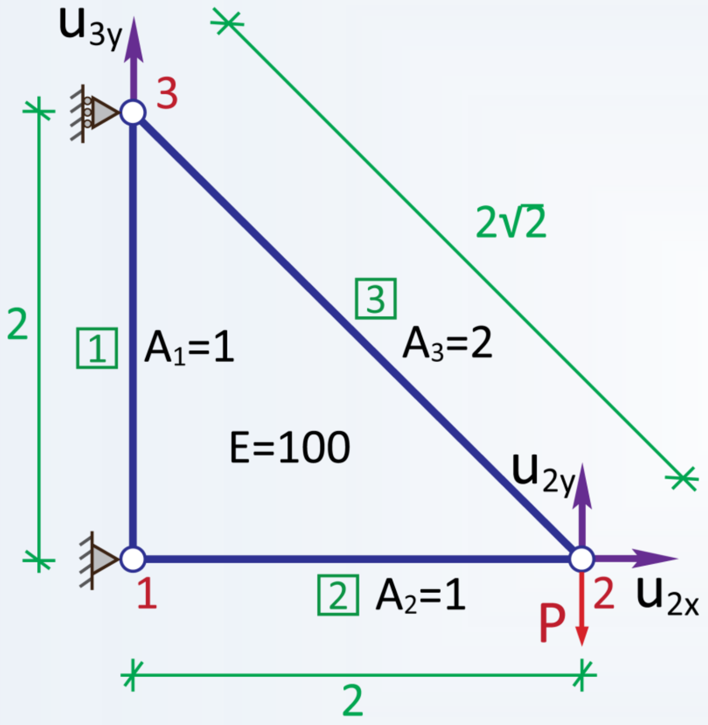
$\mathbf{u}_1 = [0\ 0\ 0\ \mathrm{u}_{3y}]^{\mathrm{T}}$, $\mathbf{u}_2 = [0\ 0\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}]^{\mathrm{T}}$,
$\mathbf{u}_3 = [0\ \mathrm{u}_{3y}\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}]^{\mathrm{T}}$,
$\mathbf{u} = [\ \mathrm{u}_{2x}\ \mathrm{u}_{2y}\ \mathrm{u}_{3y}]^{\mathrm{T}}$

$$\mathbf{C}_1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}, \mathbf{C}_2 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}, \mathbf{C}_3 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

	θ	**d**	**B**
Bar 1	$\pi/2$	$[0\ \ 1]^{\mathrm{T}}$	$[0\ -1\ 0\ 1]$
Bar 2	0	$[1\ \ 0]^{\mathrm{T}}$	$[-1\ \ 0\ 1\ 0]$
Bar 3	$7\pi/4$	$[1\ -1]^{\mathrm{T}}/\sqrt{2}$	$[-1\ 1\ 1\ -1]/\sqrt{2}$

Example

- Three-bar truss



- Stiffness matrices of the bars

$$\boxed{\boldsymbol{K}_j = \mathbf{B}_j{}^{\mathrm{T}} \left(\frac{E a_j}{L_j} \right) \mathbf{B}_j}$$

$$\boldsymbol{K}_1 = \frac{100\times1}{2} \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix}$$

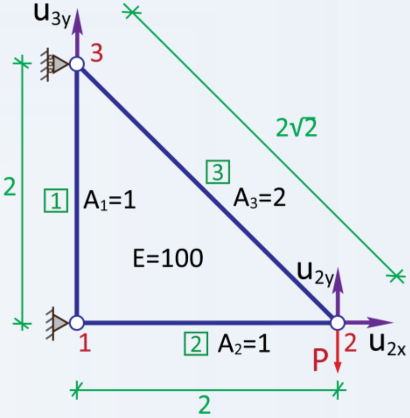
$$\boldsymbol{K}_2 = \frac{100\times1}{2} \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$\boldsymbol{K}_3 = \frac{100\times2}{4\sqrt{2}} \begin{vmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$$

- Force vectors of the bars

$\boldsymbol{f}_1 = [0\ 0\ 0\ 0]^{\mathrm{T}}$ $\quad$ $\boldsymbol{f}_2 = [0\ 0\ 0\ 0]^{\mathrm{T}}$ $\quad$ $\boldsymbol{f}_3 = [0\ 0\ 0\ \mathrm{P}]^{\mathrm{T}}$

Example



- Global versions of the element stiffness matrices and applied force vectors

$$\boxed{\boldsymbol{K}_{g,j} = \mathbf{C}_j{}^{\mathrm{T}}\boldsymbol{K}_j(a_j)\mathbf{C}_j}$$

$$\boldsymbol{K}_{g,1} = \frac{100\times 1}{2}\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \boldsymbol{K}_{g,2} = \frac{100\times 1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad \boldsymbol{K}_{g,3} = \frac{100\times 2}{4\sqrt{2}}\begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\boxed{\mathbf{f}_{g,j} = \mathbf{C}_j{}^{\mathrm{T}}\boldsymbol{f}_j}$$

$$\mathbf{f}_{g,1} = [0\ 0\ 0]^{\mathrm{T}} \qquad \mathbf{f}_{g,2} = [0\ 0\ 0]^{\mathrm{T}} \qquad \mathbf{f}_{g,3} = [0\ \mathrm{P}\ 0]^{\mathrm{T}}$$

- Global stiffness matrix and applied force vector

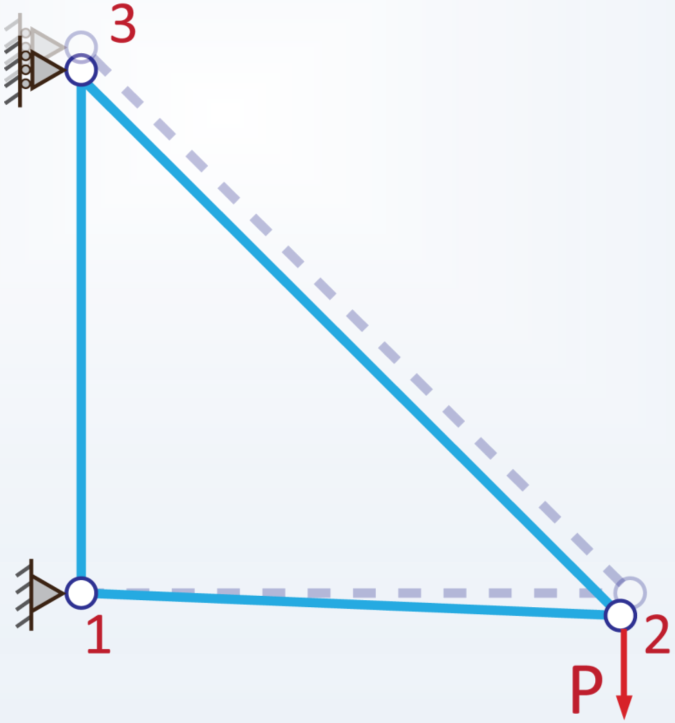
$$\mathbf{K} = \sum_{j=1}^{3}\boldsymbol{K}_j = 50\begin{vmatrix} 1+1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 1+1/\sqrt{2} \end{vmatrix} \qquad \mathbf{f} = \sum_{j=1}^{3} C_j{}^{\mathrm{T}}\mathbf{f}_{g,j} = \begin{bmatrix} 0 \\ \mathrm{P} \\ 0 \end{bmatrix}$$

Example

- Solution of the equilibrium condition

$$\boxed{\mathbf{u} = \mathbf{K}^{-1}\mathbf{f}}$$

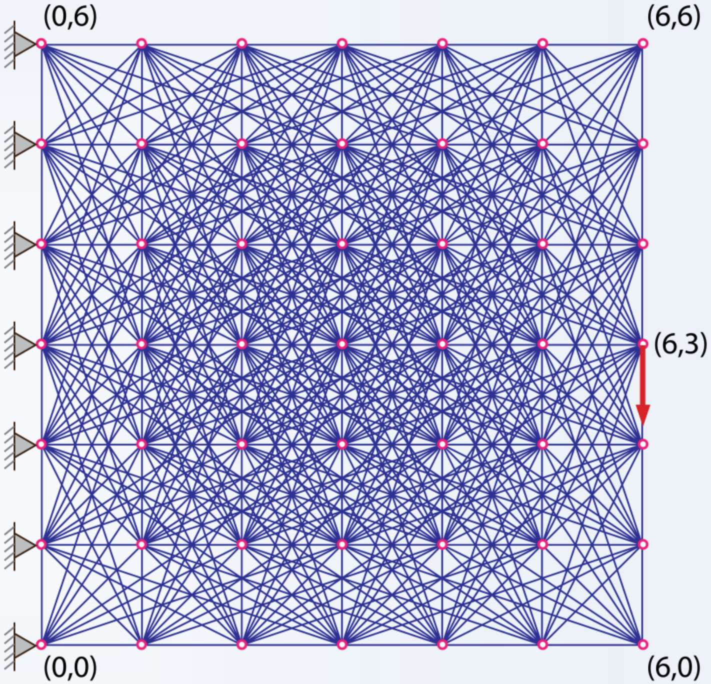
$$\mathbf{u} = [u_{2x}\ u_{2y}\ u_{3y}]^T = P/50 \times [1\ \ 2+\sqrt{2}\ \ 1]^T$$



Optimization : Example

- Optimize the ground structure
  - Material has $E = 10^7$
  - Load is $P = 1$

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{f}^{\mathrm{T}}\mathbf{u}(\mathbf{a}) \\ \text{s.t.} \quad & \sum_{j=1}^{nl} a_j L_j \leq V_{\max} \\ & a_j \geq 0 \\ \text{with} \quad & \mathbf{K}(\mathbf{a})\mathbf{u}(\mathbf{a}) = \mathbf{f} \end{aligned}$$

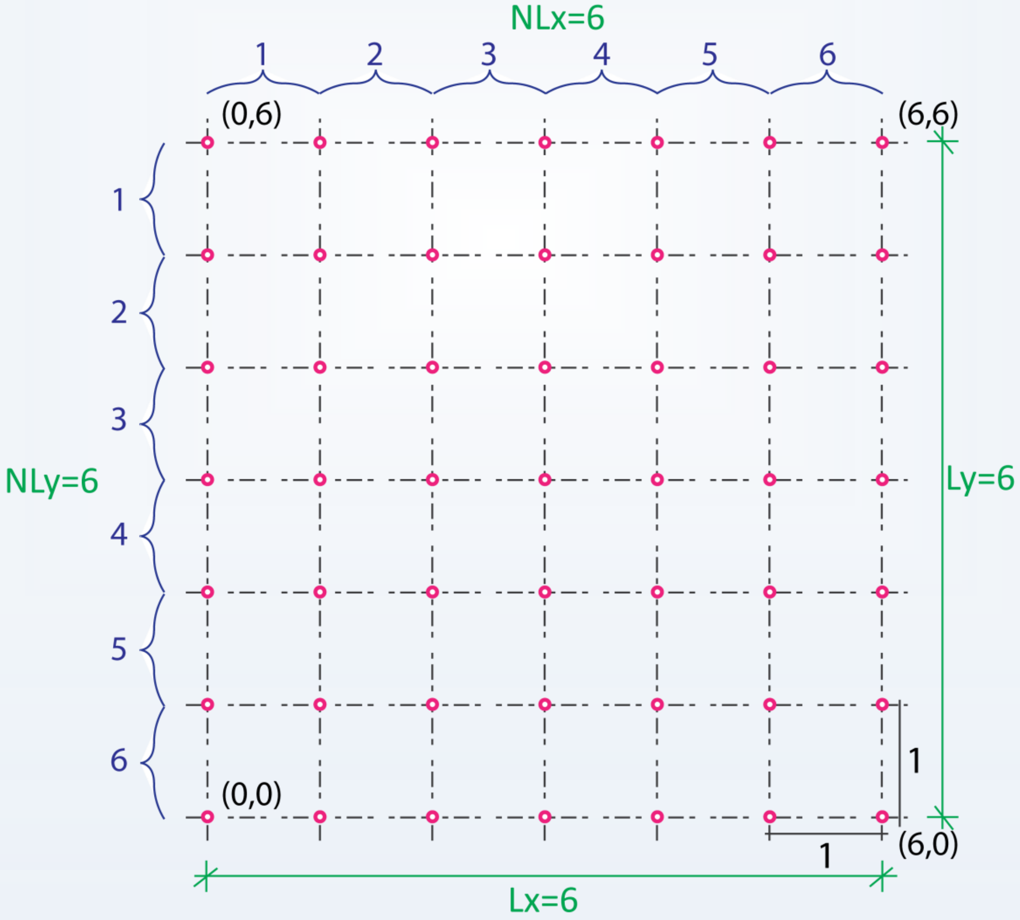


- Steps required to run `TrussTop.m`
  1. Generate design domain (initial mesh)
  2. Optimization parameters in `Ex3_Script.m`
  3. Run!

Optimization : Example

- Design domain

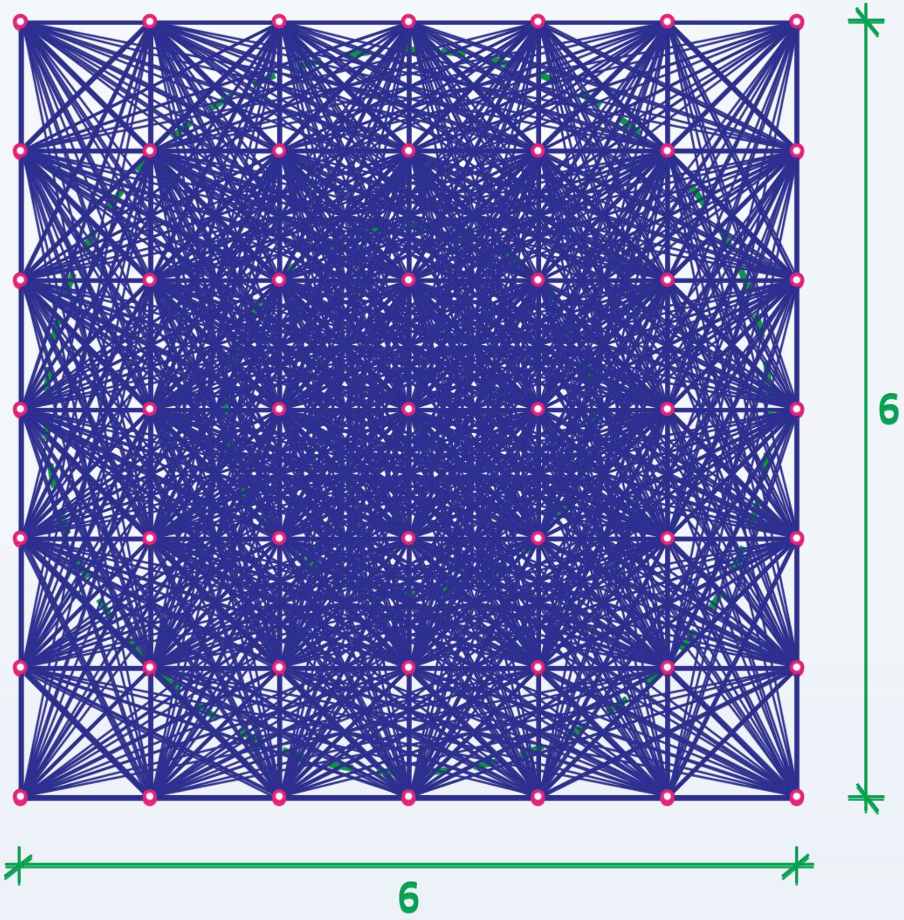
```
Lx=6; Ly=6; NLx=6; NLy=6;
```



Optimization : Example

- Initial mesh

```
Ratio = 6*sqrt(2);   % Level 6
```



Optimization : Example

- Boundary definition

```
CoordS = [restr.x restr.y coord.x coord.y];
```

(6,3)

(3,1)

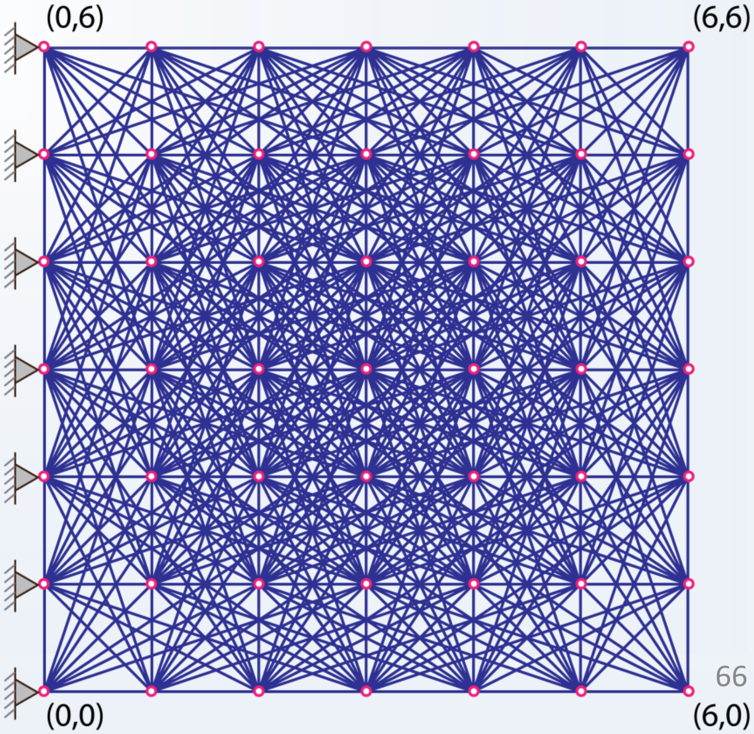
```
CoordS =[1 1 6 3]
```

```
CoordS =[0 1 3 1]
```

```
CoordS = [ 1 1 0 0;
           1 1 0 1;
           1 1 0 2;
           1 1 0 3;
           1 1 0 4;
           1 1 0 5;
           1 1 0 6];
```

or

```
CoordS = [ 1 1 0 -1];
```



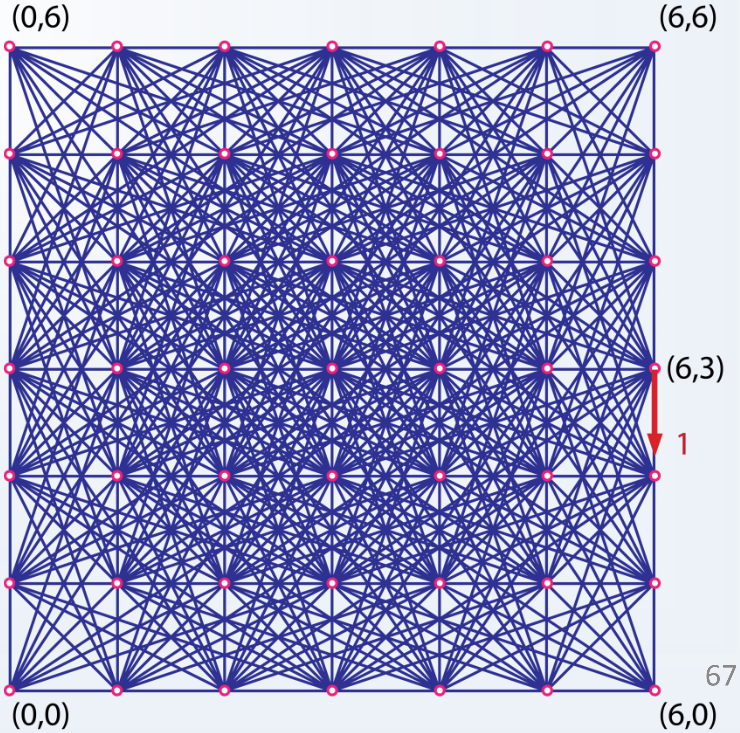
Optimization : Example

- Load definition

```
CoordL = [px py coord.x coord.y];
```

```
CoordL =[0 2 6 3]    CoordL =[1 0 3 1]
```

```
CoordL = [0 -1 6 3];
```



Optimization : Example

Young’s Modulus

Design domain size and the number of grids

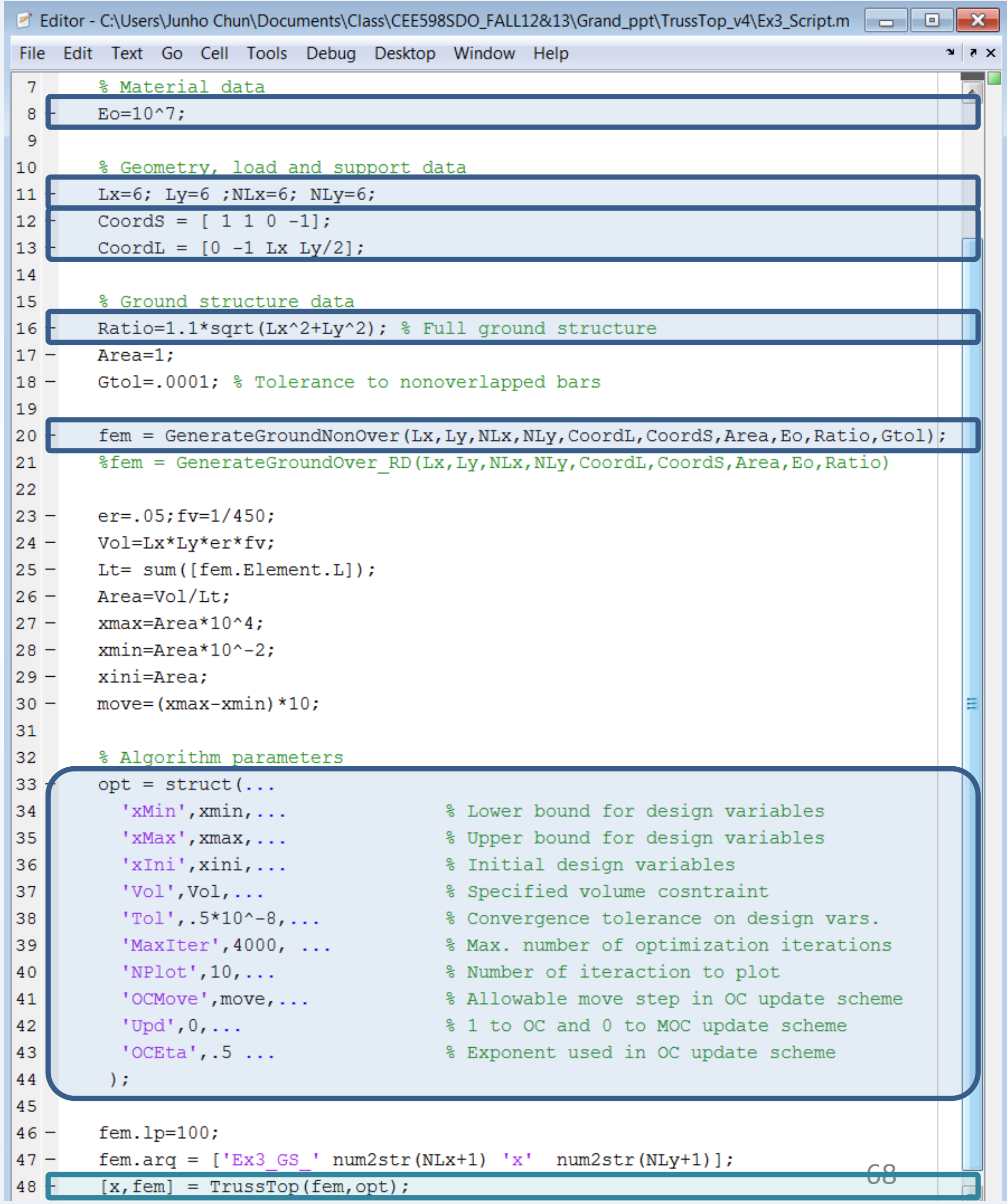
Supports and loads definition

Level of initial mesh

Mesh generator call

Optimization parameters

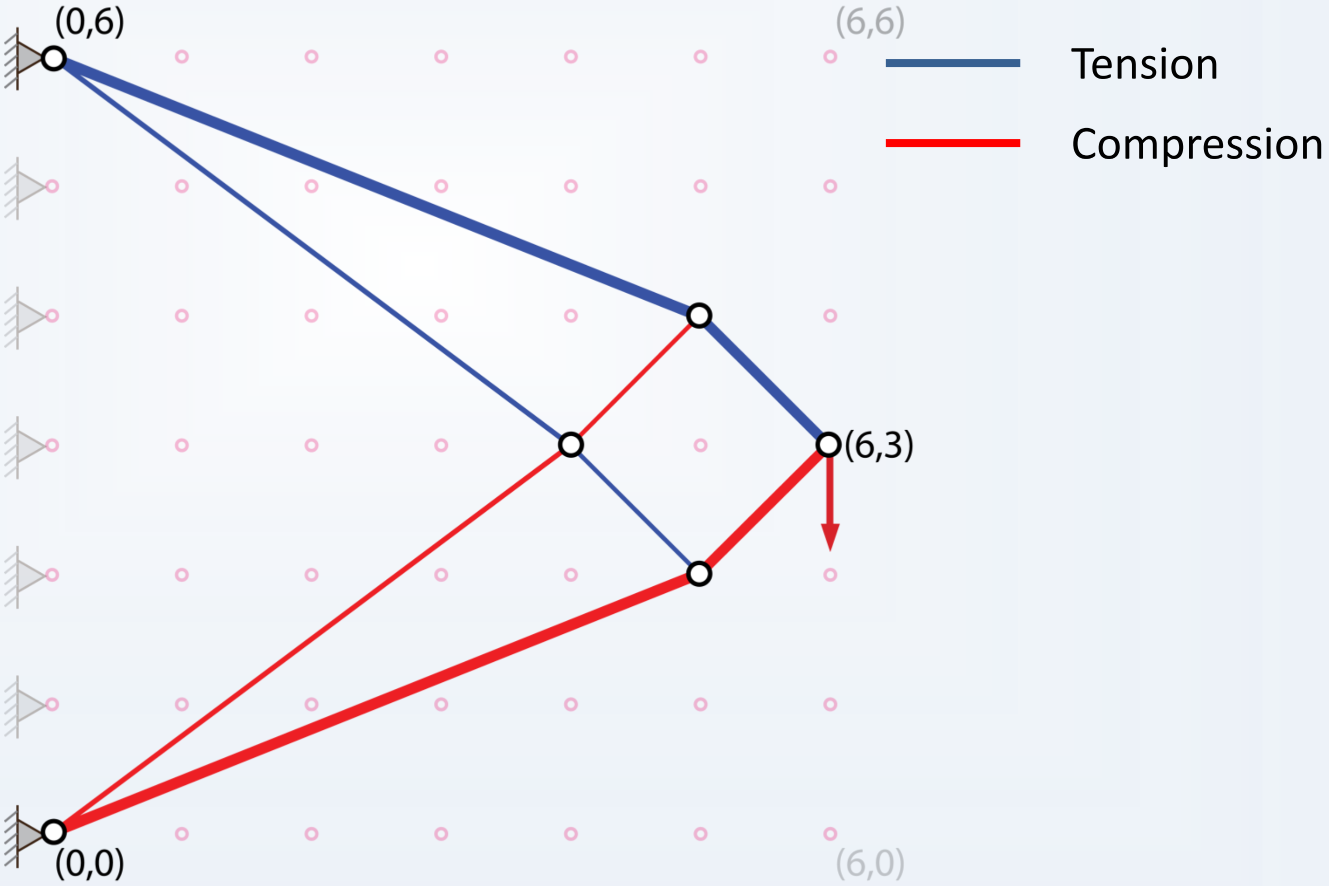
Run TrussTop



```
% Material data
Eo=10^7;

% Geometry, load and support data
Lx=6; Ly=6 ;NLx=6; NLy=6;
CoordS = [ 1 1 0 -1];
CoordL = [0 -1 Lx Ly/2];

% Ground structure data
Ratio=1.1*sqrt(Lx^2+Ly^2); % Full ground structure
Area=1;
Gtol=.0001; % Tolerance to nonoverlapped bars

fem = GenerateGroundNonOver(Lx,Ly,NLx,NLy,CoordL,CoordS,Area,Eo,Ratio,Gtol);
%fem = GenerateGroundOver_RD(Lx,Ly,NLx,NLy,CoordL,CoordS,Area,Eo,Ratio)

er=.05;fv=1/450;
Vol=Lx*Ly*er*fv;
Lt= sum([fem.Element.L]);
Area=Vol/Lt;
xmax=Area*10^4;
xmin=Area*10^-2;
xini=Area;
move=(xmax-xmin)*10;

% Algorithm parameters
opt = struct(...
  'xMin',xmin,...                 % Lower bound for design variables
  'xMax',xmax,...                 % Upper bound for design variables
  'xIni',xini,...                 % Initial design variables
  'Vol',Vol,...                   % Specified volume cosntraint
  'Tol',.5*10^-8,...              % Convergence tolerance on design vars.
  'MaxIter',4000, ...             % Max. number of optimization iterations
  'NPlot',10,...                  % Number of iteraction to plot
  'OCMove',move,...               % Allowable move step in OC update scheme
  'Upd',0,...                     % 1 to OC and 0 to MOC update scheme
  'OCEta',.5 ...                  % Exponent used in OC update scheme
 );

fem.lp=100;
fem.arg = ['Ex3_GS_' num2str(NLx+1) 'x'  num2str(NLy+1)];
[x,fem] = TrussTop(fem,opt);
```

Optimization : Example

- Optimal topology



Questions? Comments?