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Effect of material gradation on *K*-dominance of fracture specimens

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Abstract

This work describes the effect of material gradation (parallel to the crack plane) on stress intensity factors and *K*-dominance, i.e. the dominance of the singular region, of fracture specimens; SE(T), SE(B) and C(T). The extent of *K*-dominance is investigated by comparing the actual stress field with the Williams' asymptotic stress field. Linear-elastic finite element analyses are performed using graded elements which incorporate graded material properties at the element level. Material gradation and crack geometry are systematically varied to perform the parametric study. Results reveal that the effect of material gradation on K_I is most pronounced when a short crack is located on the stiffer side of the fracture specimen. For a given specimen and crack geometry, the extent of *K*-dominance yields a curve with a peak point at a certain material gradation. Results of the present study provide valuable insight into the *K*-dominance of FGMs.

Keywords: Functionally graded material (FGM); *K*-dominance; Stress intensity factor; Fracture specimens; 3-D finite element analysis; Graded element

1. Introduction

Crack-tip stress fields in graded nonhomogeneous materials, e.g. functionally graded materials (FGMs) have been investigated by many researchers [1]. Eischen [2] adopted the eigenfunction technique to determine the leading terms of the asymptotic stress field in FGMs. He observed that the asymptotic stress field near the crack-tip in a FGM is identical to that in a homogeneous material, i.e. the Williams' [3] solution. However, as the distance from the crack-tip increases, the higher-order terms are affected by the material gradation. The material gradation does not affect the order of singularity and the angular functions of the crack-tip fields, but does affect the stress intensity factors (SIFs). It also significantly affects the extent of dominance of the singular region, i.e. the *K*-dominant region. This important aspect is the subject of this work.

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Studies have been performed to investigate the *K*-dominance of FGMs. Gu and Asaro [4] studied crack deflection in FGMs where the crack plane is perpendicular to the material gradient direction. They showed that the size of the *K*-dominant region decreases as the severity of the material nonhomogeneity increases. Marur and Tippur [5] investigated the *K*-dominance in FGMs using single-edged tension, SE(T) specimens with crack perpendicular to the material gradient. They argued that the homogeneous, asymptotic solution is not valid for FGMs and suggested that higher-order terms of the series must be considered to obtain good agreement between the analytical and numerical results. Recently, Anlas et al. [6] explored the extent and the shape of the *K*-dominant region in SE(T) FGM specimens with crack parallel to the material gradient. They identified the relationship between the two different forms of asymptotic stress fields (i.e. the Williams [3] form and the modified Erdogan [7] form) and investigated the extent of validity of these fields. However, they did not appear to reach any definite conclusion. Recently, Shim et al. [8] have determined the *K*-dominance of SE(T) specimens considering various material gradations and crack geometries.

In this work, we investigate the effect of material gradation on mode I SIF, K_I, for various fracture specimens. Material gradation and crack geometry are systematically varied. Then we explore the extent of *K*-dominance for FGM fracture specimens by comparing the actual stress field with the Williams' asymptotic stress field. In the present study, the linear-elastic material properties are graded parallel to the crack plane.

2. Effect of material gradation on stress intensity factors

The present study considers the following fracture specimens:

- single-edged tension, SE(T),
- single-edged bending, SE(B),
- compact tension, C(T).

Fig. 1 shows the geometries and the dimensions of the fracture specimens, where a, w, h and t denote the crack length, specimen width, specimen height and specimen thickness, respectively. Four different crack lengths, i.e. $a/w = 0.1$, 0.3, 0.5, 0.7, are considered for each type of specimen. For all specimens, $w = 5$ (consistent units) and $t/w = 0.01$. Fig. 2 shows the three-dimensional (3-D) finite element (FE) meshes for SE(T), SE(B) and C(T) specimens. Symmetry conditions permit modelling of only one half of the specimens. The mesh consists of triquadratic (20-noded brick) elements with reduced ($2 \times 2 \times 2$) integration and has a single

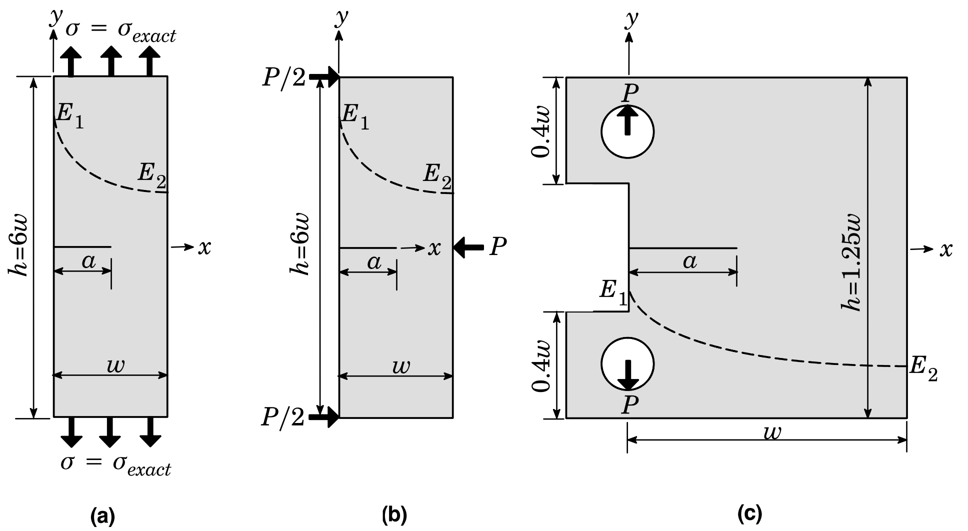

Fig. 1. Fracture specimens considered in the present study: (a) single-edged tension, SE(T); (b) single-edged bending, SE(B); (c) compact tension, C(T). Note that the specimen thickness $t = 0.01w$ for all specimens. Material properties are graded in the x-direction.

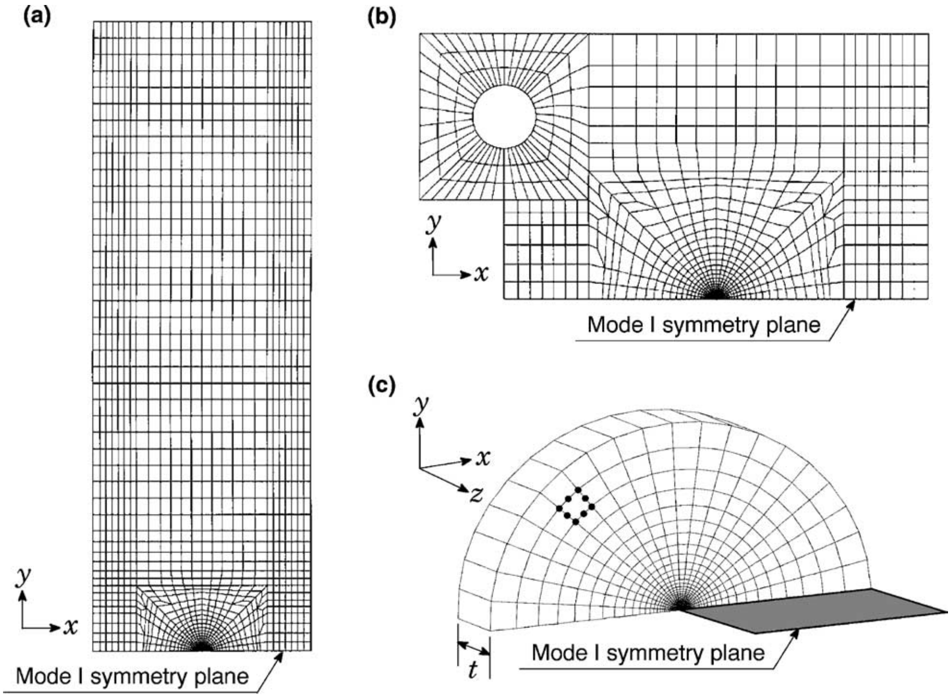

Fig. 2. Typical finite element meshes used in the present study: (a) SE(T) and SE(B); (b) C(T); (c) close-up view of the circular domain (67 rings) at the crack-tip.

layer of elements through the thickness. All nodes in the model are constrained in the thickness direction to obtain plane-strain conditions. Elements with quarter-point nodes and collapsed faces are used to model the crack-tip. The FE model for SE(T) and SE(B) specimens have 18,666 nodes and 2574 elements and C(T) specimen has 13,056 nodes and 1794 elements.

The Poisson's ratio ν is a constant (=0.3) and Young's modulus E is an exponential function

$$E(x) = E_1 e^{\lambda x}, \quad (1)$$

where λ is the material nonhomogeneity parameter and $1/\lambda$ denotes the length scale of the material. Material gradients range from $E_2/E_1 = 1/20$ to $E_2/E_1 = 20$.

Kim and Paulino [9] presented a generalized isoparametric formulation (GIF) to calculate the elastic properties within an element. Such *graded elements* include the gradation effect at the element level and thus can substantially improve the solution quality based on the same mesh density. Within graded elements, the calculation of stiffness, stress and other quantities requires the value of properties at integration points. With nodal values of material properties defined at each nodes, interpolation using element shape functions determines property values at integration points. The current study employs the nodal-value approach [9].

For graded nonhomogeneous SE(T) specimen, the uniform traction (σ) applied to the model may not be equivalent to the far-field tension—due to the material gradient in the x-direction. Kim and Paulino [9] give the exact traction (σ_{exact}) solutions equivalent to the far-field tension for exponentially graded nonhomogeneous materials. In this study, for accuracy, the traction applied to the SE(T) specimen is determined from those solutions [9].

The numerical solutions are generated using WARP3D [10], a research code for nonlinear fracture mechanics. It employs an incremental-iterative, implicit formulation for analyses of fracture models subjected to quasi-static and dynamic loading. Besides the conventional solid elements for homogeneous materials, this code also incorporates solid elements with graded elastic and plastic properties. Moreover, WARP3D incorporates the interaction integral technique to calculate SIFs and T-stresses in 3-D homogeneous [11] and func-

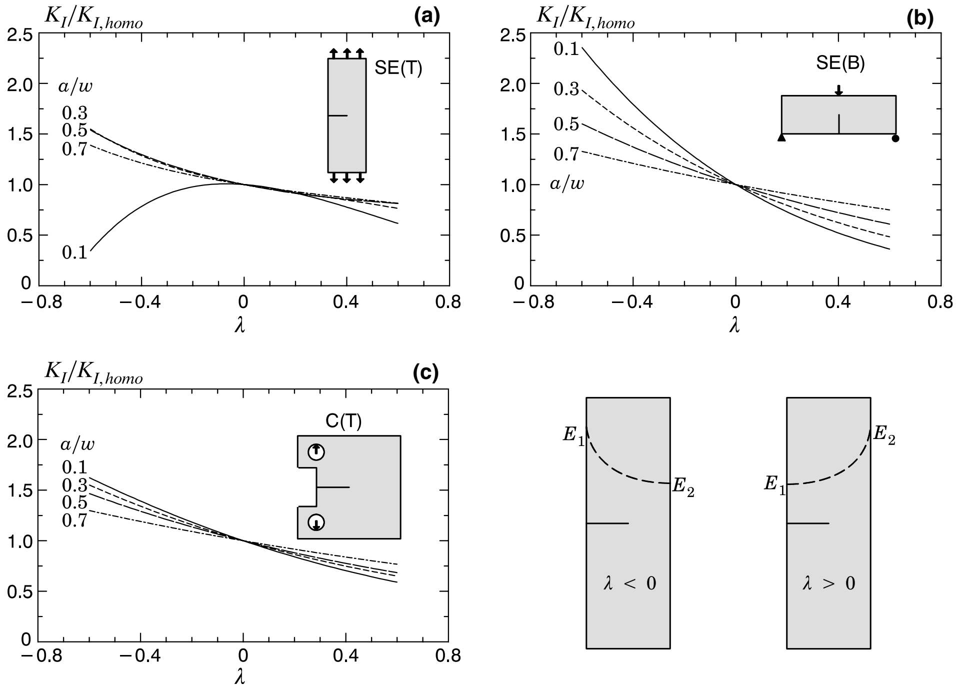

Fig. 3. Effect of material gradation on K_{I} for SE(T), SE(B) and C(T) specimens with various crack lengths.

tionally graded materials [12]. In the present study, this technique is employed to calculate the K_{I} values for the fracture specimens.

Fig. 3 shows the effect of material gradation on mode I SIF K_{I} for SE(T), SE(B) and C(T) specimens with various crack lengths (a/w). K_{I} is normalized by $K_{\mathrm{I,homo}}$, which is the K_{I} value for homogenous materials. This normalized value is plotted against the material nonhomogeneity parameter $\lambda = \ln(E_2/E_1)/w$. As the material nonhomogeneity increases, i.e. as the absolute value of λ increases, the difference between K_{I} and $K_{I,\mathrm{homo}}$ increases. When the crack is located on the stiffer side of the specimen ($E_1 > E_2$), the K_{I} value of the nonhomogeneous material is larger than that of the homogeneous material—except for SE(T) specimen with a short crack ($a/w = 0.1$), where the deformation of the specimen due to the material gradation reduces the K_{I} value. Similar results for short cracks in FGM SE(T) specimens have been presented in Ref. [13]. On the other hand, when the crack is located on the softer side of the specimen ($E_1 < E_2$), the K_{I} value of the nonhomogeneous material is smaller than that of the homogeneous material. However, the difference between K_{I} and $K_{I,\mathrm{homo}}$ is smaller than the case where $E_1 > E_2$. For a specific value of λ , the effect of the material gradation on K_{I} is more intense for shorter cracks for all specimens. Fig. 3 also shows that the effect of material gradation on K_{I} is most pronounced for SE(B) specimen.

3. *K*-dominance of FGM fracture specimens

In this section, we investigate the extent of *K*-dominance for FGM fracture specimens. This is achieved by comparing the actual crack-tip stress field with the Williams' [3] asymptotic stress field. Elastic crack-tip stress fields are obtained from the nodal stress values and the asymptotic stress fields are obtained from the K_{I} values calculated from the previous section. Since the stress in the y-direction, σ_{yy}, is the dominant stress, we define the *K*-dominant region by comparing σ_{yy} at the symmetry plane ($y = 0$). *The K-dominant region is defined as the region where the actual and asymptotic stresses differ within 5%* [8].

Fig. 4 shows the extent of *K*-dominance for FGM fracture specimens considering crack geometry and material gradation. The range of *K*-dominant region ($r/a \times 100$) is plotted against the normalized material nonho-

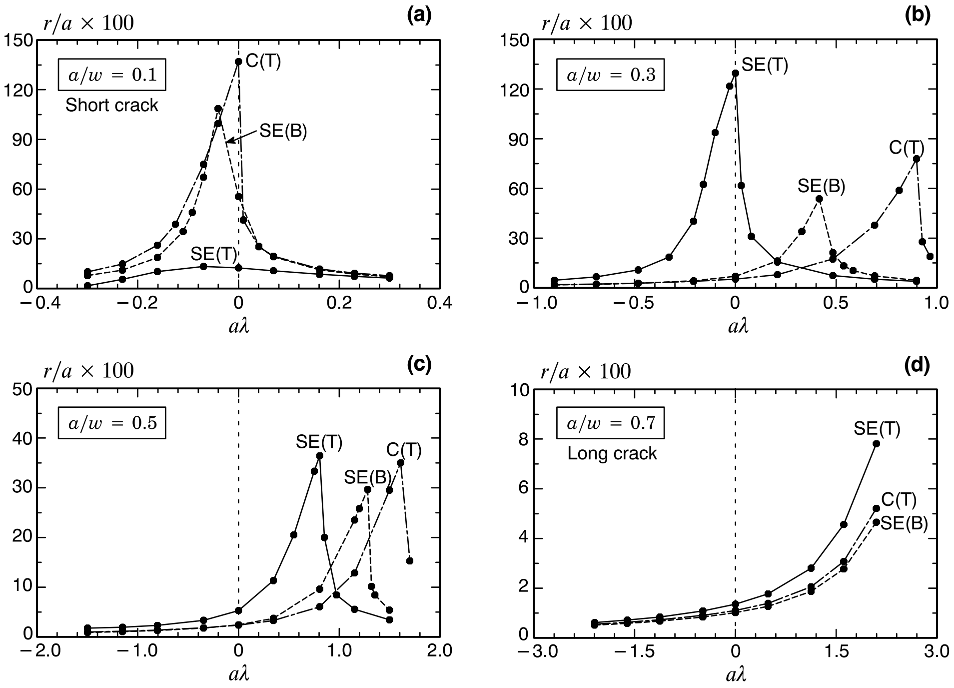

Fig. 4. Extent of *K*-dominance for SE(T), SE(B) and C(T) specimens considering material gradation and crack geometry.

mogeneity parameter, $a\lambda = \ln(E_2/E_1)a/w$, which represents the material gradation. Note that r is the distance from the crack-tip on the crack plane. *Fig. 4 demonstrates that the material gradation and crack geometry have significant effect on the extent of K-dominance*. For a given specimen and crack geometry, the extent of *K*-dominance yields a curve with a peak point at a certain material gradation. This is due to the fact that the higher-order terms of the actual stress fields are affected by both the crack geometry and the material gradation, which interact in a complex way. For C(T) specimen with $a/w = 0.3$ and 0.5, additional range of material gradation ($20 \leqq E_2/E_1 \leqq 30$) is considered to obtain the peak point. For some cases considered in the present study, i.e. C(T) with $a/w = 0.1$ and SE(T) with $a/w = 0.3$, homogeneous material yields the maximum extent of *K*-dominance. However, Fig. 4 indicates that the maximum extent of *K*-dominance does not always prevail for homogenous materials. Note that, for some cases, the extent of *K*-dominance increases as the material nonhomogeneity increases. The extent of *K*-dominance is relatively small for all specimens when $a/w \geqq 0.3$ (for SE(T), $a/w \geqq 0.5$) and crack-tip is located on the stiffer side of the specimen ($E_1 > E_2$). Results for SE(B) and C(T) specimens are very similar when $a/w = 0.1$, 0.7 and when $a/w = 0.3$, 0.5 with $E_1 > E_2$.

4. Concluding remarks

The effect of material gradation on SIFs for fracture specimens, i.e. SE(T), SE(B) and C(T) specimens, with various crack lengths has been investigated. Results reveal that the effect of material gradation on K_{I} is most pronounced when a short crack is located on the stiffer side of the fracture specimen. Among the three specimens considered in the present study, K_I for SE(B) specimens are most highly affected by the material gradation. A parametric study provides the extent of *K*-dominance for FGM fracture specimens. For a given specimen and crack geometry, the extent of *K*-dominance yields a curve with a peak point at a certain material gradation. The size of the *K*-dominant region does not always decrease as the severity of the material nonhomogeneity increases, but in some cases, the extent of *K*-dominance increases as the material nonhomogeneity increases. The present results provide valuable insights into the *K*-dominance of FGMs considering material gradation and crack geometry. These results also serve as a guideline for applying the modified boundary layer model [8] to FGMs.

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