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Parameter sensitivity of system reliability using sequential compounding method

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ABSTRACT

Computation of sensitivities of the 'system' failure probability with respect to various parameters is essential in reliability based design optimization (RBDO) and uncertainty/risk management of a complex engineering system. The system failure event is defined as a logical function of multiple component events representing failure modes, locations or time points. Recently, the sequential compounding method (SCM) was developed for efficient calculations of the probabilities of large-size, general system events for a wide range of correlation properties. To facilitate the use of SCM in RBDO and uncertainty/risk management under a constraint on the system failure probability, a method, termed as Chun–Song–Paulino (CSP) method, is developed in this paper to compute parameter sensitivities of system failure probability using SCM. For a parallel or series system, the derivative of the system failure probability with respect to the reliability index is analytically derived at the last step of the sequential compounding. For a general system, the sensitivity of the probability of the set involving the component of interest and the sensitivity of the system failure probability with respect to the super-component representing the set are computed respectively using the CSP method and combined by the chain-rule. The CSP method is illustrated by numerical examples, and successfully tested by examples covering a wide range of system event types, reliability indices, number of components, and correlation properties. The method is also applied to compute the sensitivity of the first-passage probability of a building structure under stochastic excitations, modeled by use of finite elements.

1. Introduction

Sensitivity analysis is an important part of determining impacts of input variables on the function, system or performance output. Such an analysis not only provides quantitative measures that help identify relative importance of variables in terms of their impact on the results, but also facilitate the use of gradient-based optimizers in efforts to optimize the system. In risk-based decision making processes to improve or optimize a system subjected to significant uncertainties, it is essential to identify relative contributions of various input random variables in terms of parameter sensitivities of the failure probability. To this end, various sensitivity-based importance measures have been developed. Such measures quantify relative importance of random variables in terms of the difference in the failure probability caused by the changes in the distribution parameters proportional to the standard deviations or those made possible by the fixed upgrade cost [1,2].

The recent emergence of research in reliability based design optimization (RBDO) [3–8] also demands calculating parameter sensitivity of the failure probability. In fact, RBDO aims to find the values of design variables that maximize or minimize a given objective function describing the performance of the system while satisfying probabilistic constraints. A typical RBDO formulation is

$$\begin{aligned} \min_{\mathbf{d}} \quad & f_{\text{obj}}(\mathbf{d}) \\ \text{s.t.} \quad & P(E_i;\mathbf{d}) \leqslant P_i^{\text{target}},\ i = 1,\ldots,n_c \\ & \mathbf{d}^{\text{lower}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\text{upper}} \end{aligned} \quad (1)$$

where $f_{\text{obj}}(\mathbf{d})$ is the objective function of a given design optimization problem, e.g., volume, total cost and performance measure, $\mathbf{d} = \{d_1,\ldots,d_n\}$ is the set of the design variables with the lower bounds $\mathbf{d}^{\text{lower}}$ and the upper bounds $\mathbf{d}^{\text{upper}}$ (box constraints), and

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E_i and P_i^{target} respectively denote the event that the i-th constraint is violated, $i = 1, \ldots, n_c$ (or the i-th failure event), and the corresponding target failure probability. Sensitivity analysis of the probabilistic constraints with respect to design variables is a crucial part of the reliability based design optimization especially when a gradient-based optimization algorithm needs to be utilized.

In the aforementioned situations of RBDO, if the failure event is described as a system event E_{sys}, i.e., a logical function of multiple component events representing failure modes, locations or time points, parameter sensitivities of the system failure probability are needed. Among various examples of system failure events [9,10], let us consider the first passage probability of a structure subject to stochastic excitations [11–15]. This is the probability that a stochastic response $X(t)$ exceeds a given threshold x_0 at least once for a given duration $t \in (0, t_n]$. This is commonly utilized to find the probability of the failure event described within a time interval. One of the available approaches for formulating the first passage probability consists of defining the problem as a series system problem [13], i.e.,

$$P(E_{\text{sys}}) = P(x_0 < \max_{0<t\leqslant t_n} |X(t)|) = P\left(\bigcup_{i=1}^{n} |X(t_i)| > x_0\right) \tag{2}$$

where t_i is the i-th discretized time point, $i = 1,\ldots,n$. The first passage probability defined in Eq. (2) requires evaluation of the component failure probability at each time point and the statistical dependence between the failures at different time points. If a probabilistic constraint is associated with the first passage probability in RBDO [15], an efficient, reliable and robust algorithm is required to compute the system failure probability during the iterative procedure in RBDO.

In general, the sensitivity of the system failure probability with respect to a parameter θ is obtained by a chain rule, i.e.,

$$\frac{\partial P(E_{\text{sys}})}{\partial \theta} = \sum_{i=1}^{n} \frac{\partial P(E_{\text{sys}})}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial \theta} \tag{3}$$

where β_i is the reliability index of the i-th component failure event. It is noted that the impact of the correlation between component failure events on the partial derivative is assumed to be negligible. From Eq. (3), the partial derivatives of the component reliability index with respect to the design variables are available from parameter sensitivities of component reliability analysis [1,16]. However, the derivative of the system failure probability with respect to the reliability index has not yet been clearly addressed in the literature. Several methods have been developed to compute parameter sensitivities of the system failure probability. Song and Kang [10] used the Matrix-based System Reliability method [17] for computing parameter sensitivities for systems under statistical dependence, and later the method was further developed [18]. In [10] and [18], the sensitivity of the system failure probability was computed with respect to the mean and the standard deviation of the input random variables to facilitate the decision-making process and system reliability-based design optimization [7,19]. Sensitivity-based importance measures [1,2] were also computed to quantify the relative importance of the design variables. Sues and Cesare [20] proposed a method of computing parameter sensitivity of the system failure probability using the results of component reliability analysis by the first-order reliability method and Monte Carlo simulations. Song and Der Kiureghian [21] utilized the linear programing bounds method [9] in order to compute lower and upper bounds of the parameter sensitivities of general system events, even with incomplete information on component probabilities and their statistical dependence. Despite these proposed methods, computing parameter sensitivities of the system failure probability is still challenging if the system has a large number of components and/or the correlation properties of component failure events do not allow for achieving conditional independence between components given a small number of common source random variables [18].

Therefore, in this paper, a method of computing parameter sensitivity of the system failure probability is proposed using the sequential compounding method (SCM) [22] which was recently developed to compute multivariate normal integrals of general system events with a wide range of correlation properties even for those with a large number of component events. The proposed method, termed as Chun–Song–Paulino (CSP) method, is illustrated and tested by a variety of numerical examples. The CSP method is further demonstrated by application to the first passage probability of a structure described by a finite element model subjected to stochastic excitations.

The remainder of the paper is structured as follows. After a brief summary of the SCM [22], the SCM-based parameter sensitivity formulations are derived for series, parallel and general systems (cut-set system) respectively. Numerical examples test the CSP method and demonstrate its application to first-passage problems. Finally, concluding remarks and discussions on future research needs are provided.

2. Sequential compounding method

In the sequential compounding method (SCM; Kang and Song [22]), two component events coupled by a union or intersection operation in the system event are compounded sequentially until a single compound event eventually represents the system event. Each compounding procedure consists of determining the probability of the new compound event, and evaluating the correlation coefficient between the new compound event and each of the remaining component events.

First, when two events are coupled by an intersection operation, the compounding process starts by obtaining the reliability index of the compound event $E_{\text{iandj}} = E_i \cap E_j$ as

$$\beta_{\text{iandj}} = -\Phi^{-1}[P(E_i \cap E_j)] = -\Phi^{-1}[\Phi_2(-\beta_i, -\beta_j; \rho_{i,j})] \tag{4}$$

where β_{iandj} denotes the reliability index of the compound event, $\Phi(\cdot)$ is the marginal cumulative distribution function (CDF) of the standard normal distribution, and $\Phi_2(\cdot)$ is the joint CDF of the bivariate standard normal distribution. $\rho_{i,j}$ is the correlation coefficient between the standard normal random variables representing E_i and E_j, which could be obtained from the inner-product of the negative normalized vectors of the design points [23]. After β_{iandj} is obtained, the correlation coefficient between E_{iandj} and each of the remaining component events $E_k, k = 1, \ldots, n, k \neq i, j$, denoted by $\rho_{(\text{iandj}),k}$, is computed. The correlation coefficient is determined such that the compound event can represent $E_i \cap E_j$ accurately in computing the probability of $E_i \cap E_j \cap E_k$, i.e.,

$$\Phi_3(-\beta_i, -\beta_j, -\beta_k; \rho_{i,j}, \rho_{i,k}, \rho_{j,k}) = \Phi_2(-\beta_{\text{iandj}}, -\beta_k; \rho_{(\text{iandj}),k}) \tag{5}$$

In Eq. (5), $\rho_{(\text{iandj}),k}$ is the only unknown variable, which can be obtained numerically by nonlinear programing

$$\begin{aligned} &\min_{\rho_{(\text{iandj}),k}} \quad \left|\Phi_3(-\beta_i, -\beta_j, -\beta_k; \rho_{i,j}, \rho_{i,k}, \rho_{j,k}) - \Phi_2(-\beta_{\text{iandj}}, -\beta_k; \rho_{(\text{iandj}),k})\right| \\ &\text{s.t.} \quad -1 \leqslant \rho_{(\text{iandj}),k} \leqslant 1 \end{aligned} \tag{6}$$

The multi-fold integrals of the joint CDFs in the optimization problem can be performed by efficient algorithms such as the one by Genz [24]. To further reduce the computational costs for solving Eq. (5) during the optimization process, Kang and Song [22] proposed an approximate procedure as well, which deals with single-fold integrals only.

Similarly, components coupled by a union operation can be compounded as follows. The equivalent reliability index β_{iorj} is obtained by

$$\begin{aligned}\beta_{\mathrm{iorj}} &= -\Phi^{-1}[P(E_i \cup E_j)] = -\Phi^{-1}[1 - P(\overline{E}_i \cap \overline{E}_j)] \\ &= \Phi^{-1}[\Phi_2(\beta_{\mathrm{i}}, \beta_{\mathrm{j}}; \rho_{i,j})]\end{aligned} \tag{7}$$

The equivalent correlation coefficients between the compound event E_{iorj} and each of the remaining component events, $E_k, k = 1, \ldots, n, k \neq i, j$, denoted by $\rho_{(\mathrm{iorj}),k}$, is determined such that the compound event can represent $E_i \cup E_j$ accurately in computing the probability of $(E_i \cup E_j) \cap E_k$.

The efficiency, accuracy and applicability to series, parallel and general systems have been successfully demonstrated by numerical examples [22]. These examples cover a wide range of system event types, component probability levels, number of components, and correlation properties. In one of the examples, the failure probability of a general (cut set) system consisting up to 1000 component events was computed accurately and efficiently using the SCM.

3. Parameter sensitivity of system failure probability using SCM

To allow efficient and accurate calculations of parameter sensitivities for general large-size system events and a wide range of correlation properties, the following sensitivity formulations are developed for series, parallel and cut-set system events, respectively.

3.1. Series systems

First consider a series system event E_{series} consisting of n component events. The failure probability of the series system can be formulated as a multinormal integral problem

$$\begin{aligned}P(E_{\mathrm{series}}) &= P(E_1 \cup E_2 \cup \cdots \cup E_n) = P\left[\bigcup_{j=1}^{n}(Z_j \leqslant -\beta_j)\right] \\ &= \int_{\Omega} \varphi_n(\mathbf{z}; \mathbf{R}) d\mathbf{z}\end{aligned} \tag{8}$$

where $Z_j, j = 1, \ldots, n$, is the standard normal random variable representing E_j, $\mathbf{R}$ is the correlation coefficient matrix of $\mathbf{Z} = \{Z_1, \ldots, Z_n\}$, and $\varphi_n(\mathbf{z}; \mathbf{R})$ is the joint probability density function (PDF) of the standard normal random variables with $\mathbf{R}$. For a parameter sensitivity calculation in Eq. (3), the sensitivity of the system failure probability needs to be calculated with respect to the reliability index of the k-th event E_k, i.e.,

$$\frac{\partial P(E_{\mathrm{series}})}{\partial \beta_k} = \frac{\partial}{\partial \beta_k}\left(\int_{\Omega} \varphi_n(\mathbf{z}; \mathbf{R}) d\mathbf{z}\right) \tag{9}$$

Numerical analysis schemes are commonly implemented to solve Eq. (9) because the multi-fold integral defined in Eq. (8) and its derivative cannot be computed analytically. However, such an approach might require large computational cost or suffer numerical issues. For example, the finite difference method requires finding an admissible perturbation for accurate results.

For SCM-based calculation of sensitivities of $P(E_{\mathrm{series}})$ with respect to β_k, all components in the series system except E_k are first compounded to E_{S_k}, i.e.,

$$E_{S_k} = \bigcup_{p \in S_k} E_p \tag{10}$$

where S_k denotes the index set of the components in the series system except k, i.e., $S_k = \{1, 2, \ldots, (k-1), (k+1), \ldots, n\}$. The sequential compounding process would be completed by compounding E_k and E_{S_k}, i.e.,

$$\begin{aligned}P(E_{\mathrm{series}}) &= P(E_k \cup E_{S_k}) \\ &= P(E_k) + P(E_{S_k}) - P(E_k \cap E_{S_k}) \\ &= \Phi(-\beta_k) + \Phi(-\beta_{S_k}) - \Phi_2(-\beta_k, -\beta_{S_k}; \rho_{k,S_k})\end{aligned} \tag{11}$$

where ρ_{k,S_k} is the updated correlation coefficient between the component E_k and the compound event E_{S_k}. From Eq. (11), the sensitivity of the system failure probability with respect to β_k is obtained as

$$\frac{\partial P(E_{\mathrm{series}})}{\partial \beta_k} = -\varphi(-\beta_k) - \frac{\partial \Phi_2(-\beta_k, -\beta_{S_k}; \rho_{k,S_k})}{\partial \beta_k} \tag{12}$$

The partial derivative of the bivariate normal cumulative distribution in the last term of Eq. (12) is then computed as followings:

$$\begin{aligned}\frac{\partial \Phi_2(-\beta_k, -\beta_{S_k}; \rho_{k,S_k})}{\partial \beta_k} &= \frac{\partial\left(\int_{-\infty}^{-\beta_k}\int_{-\infty}^{-\beta_{S_k}} \varphi_2(u, v; \rho_{k,S_k}) dv du\right)}{\partial \beta_k} \\ &= -\int_{-\infty}^{-\beta_{S_k}} \varphi_2(-\beta_k, v; \rho_{k,S_k}) dv \\ &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\beta_k^2}{2}\right) \cdot \int_{-\infty}^{-\beta_{S_k}} \frac{1}{\sqrt{2\pi}\sqrt{1-\rho_{k,S_k}^2}} \\ &\quad \times \exp\left[-\frac{\left(v + \beta_k \rho_{k,S_k}\right)^2}{2\left(1-\rho_{k,S_k}^2\right)}\right] dv\end{aligned} \tag{13}$$

Changing the variable of the integral to $v' = (v + \beta_k \rho_{k,S_k})/\sqrt{1-\rho_{k,S_k}^2}$, Eq. (13) is simplified as

$$\frac{\partial \Phi_2(-\beta_k, -\beta_{S_k}; \rho_{k,S_k})}{\partial \beta_k} = -\varphi(-\beta_k) \cdot \Phi\left[\frac{-\beta_{S_k} + \beta_k \rho_{k,S_k}}{\sqrt{1-\rho_{k,S_k}^2}}\right] \tag{14}$$

Finally, substituting Eq. (14) into Eq. (12), the sensitivity of the system failure probability is derived as

$$\frac{\partial P(E_{\mathrm{series}})}{\partial \beta_k} = -\varphi(-\beta_k)\left\{1 - \Phi\left[\frac{-\beta_{S_k} + \beta_k \rho_{k,S_k}}{\sqrt{1-\rho_{k,S_k}^2}}\right]\right\} \tag{15}$$

It is noteworthy that the proposed Chun–Song–Paulino (CSP) method allows one to compute the parameter sensitivity without additional computational cost other than the simple calculation in Eq. (15) because β_{S_k} and ρ_{k,S_k} are already available from the sequential compounding procedure to obtain $P(E_{\mathrm{series}})$. It is also seen from Eq. (15) that when the k-th component is statistically independent of the others, i.e., $\rho_{k,S_k} = 0$, the sensitivity in Eq. (15) becomes $-\varphi(-\beta_k)[1 - \Phi(-\beta_{S_k})]$. Whether the components are independent or not, the sensitivity is always negative.

3.2. Parallel systems

Similarly, for SCM-based calculation of sensitivities of the parallel system failure probability $P(E_{\mathrm{parallel}}) = P(\bigcap_{j=1}^{n} E_j)$ with respect to β_k, all components in the parallel system except E_k are first compounded to E_{P_k}, i.e.,

$$E_{P_k} = \bigcap_{p \in P_k} E_p \tag{16}$$

where P_k denotes the index set of all components in the parallel system except k, i.e., $P_k = \{1, 2, \ldots, (k-1), (k+1), \ldots, n\}$. The sequential compounding process would be completed by compounding E_k and E_{P_k}, i.e.,

$$P(E_{\mathrm{parallel}}) = P(E_k \cap E_{P_k}) = \Phi_2(-\beta_k, -\beta_{P_k}; \rho_{k,P_k}) \tag{17}$$

From Eq. (14), the sensitivity is derived as

$$\frac{\partial P(E_{\text{parallel}})}{\partial \beta_k} = -\varphi(-\beta_k)\cdot\Phi\left[\frac{-\beta_{P_k}+\beta_k\rho_{k,P_k}}{\sqrt{1-\rho_{k,P_k}^2}}\right] \tag{18}$$

When the k-th component is statistically independent of the others, i.e., $\rho_{k,P_k}=0$, the sensitivity in Eq. (18) becomes $-\varphi(-\beta_k)\Phi(-\beta_{P_k})$. Whether they are independent or not, the sensitivity is always negative.

3.3. General systems

Let us next consider the probability of a general system event described by the union of cut-sets, i.e.,

$$P(E_{\text{cut-set}}) = P\left(\bigcup_{m=1}^{n} E_{C_m}\right) = P\left[\bigcup_{m=1}^{n}\left(\bigcap_{j\in C_m} E_j\right)\right] \tag{19}$$

where C_m denotes the index set of the components that belong to the m-th cut-set. The SCM-based sensitivity of the probability in Eq. (19) with respect to β_k is computed as follows. Suppose E_k, the component event of interest for sensitivity calculation, belongs to a cut-set C_l. From the chain rule, the sensitivity of the probability of the cut-set system with respect to β_k is derived as

$$\begin{aligned}\frac{\partial P(E_{\text{cut-set}})}{\partial \beta_k} &= \frac{\partial P(E_{\text{cut-set}})}{\partial P(E_{C_l})}\cdot\frac{\partial P(E_{C_l})}{\partial \beta_k}\\ &= \left[\frac{\partial P(E_{\text{cut-set}})}{\partial \beta_{C_l}}\cdot\frac{\partial \beta_{C_l}}{\partial P(E_{C_l})}\right]\cdot\frac{\partial P(E_{C_l})}{\partial \beta_k}\\ &= -\frac{1}{\varphi(-\beta_{C_l})}\cdot\frac{\partial P(E_{\text{cut-set}})}{\partial \beta_{C_l}}\cdot\frac{\partial P(E_{C_l})}{\partial \beta_k}\end{aligned} \tag{20}$$

In order to complete the calculation in Eq. (20), all components in the cut set C_l are first compounded to determine the reliability index of the supercomponent representing the cut-set, i.e., β_{C_l}. Next, the sensitivity of $P(E_{C_l})$ with respect to β_k is computed using the CSP method for parallel systems. Then, considering C_l as a component in a series system, the sensitivity of the system failure probability with respect to β_{C_l} is computed using the CSP method for series systems. These are substituted to Eq. (20) to compute the sensitivity. A similar procedure can be derived for a link-set system event, which is the intersection of unions.

4. Numerical examples

The CSP method is first illustrated with small-size numerical examples of series and cut-set systems. The method is then tested by series and parallel systems with equal or unequal reliability indices and a wide range of correlation properties. As an application example, the sensitivity of the first-passage probability of a structure is computed using the CSP method. For verification purposes, the finite difference method (FDM) employing the SCM and the Monte Carlo simulation (MCS) are also carried out. Unless stated otherwise, the five sensitivity calculations by MCS use a sample size of 10^8 with perturbation sizes $\Delta\beta = 10^{-3}$ and 10^{-4}. Then, the resulting sensitivities from different perturbation values are averaged for comparison with the results obtained using the CSP method and FDM.

4.1. Illustrative example: a series system with three components

For the illustration purpose of the proposed method, consider the series system event consisting of three components, i.e.,

$$P(E_{\text{sys}}) = P(E_1 \cup E_2 \cup E_3) \tag{21}$$

Suppose that the components have unequal reliability indices, i.e., $\beta_1 = 2$, $\beta_2 = 1.5$ and $\beta_3 = 1$. The correlation coefficient matrix of the standard normal random variables representing the three components is given by

$$\mathbf{R} = \begin{bmatrix} 1 & 0.4 & 0.2\\ 0.4 & 1 & 0.4\\ 0.2 & 0.4 & 1\end{bmatrix} \tag{22}$$

Suppose that the sensitivity of the system failure probability in Eq. (21) with respect to the reliability index β_1 is of interest. Following the CSP method, two events E_2 and E_3 are compounded so that the equivalent component $E_{2\text{or}3}$ is identified. Note that E_k and E_{S_k} in Eq. (11) correspond to E_1 and $E_{2\text{or}3}$ in this example, respectively. From Eq. (7), the reliability index $\beta_{1\text{or}2}$ is obtained as 0.8471. The equivalent correlation coefficient between E_1 and $E_{2\text{or}3}$ is computed as 0.3018 using the aforementioned procedure. This means the series system event consists of two events E_1 and $E_{2\text{or}3}$, and their correlation coefficient is 0.3018. Using Eq. (15), the sensitivity with respect to β_1 is computed as -0.0324. Sensitivities with respect to the other reliability indices are also computed similarly, and are summarized in Table 1. The results are successfully verified by the FDM with a perturbation of $\Delta\beta_i = 10^{-8}$, $i = 1, 2, 3$ and MCS as shown in Fig. 1. Effects of varying perturbations from 10^{-1} to 10^{-16} in FDM and MCS on sensitivity results are tabulated in Table 2. Both FDM and MCS are sensitive to the perturbation size while it is hard to determine an optimal perturbation a priori. It is noteworthy that MCS in particular could provide fairly inaccurate results for small perturbations.

4.2. Illustrative example: a cut-set system with six components

As another illustrative example of the CSP method, consider the following cut-set system problem consisting of six components where each cut-set subsystem has three components:

$$P(E_{\text{sys}}) = P(E_{C_1} \cup E_{C_2}) = P(E_1E_2E_3 \cup E_4E_5E_6) \tag{23}$$

Table 1
CSP Sensitivity of the series system failure probability in Example 4.1.

Component	β_k	β_{S_k}	ρ_{k,S_k}	$\partial P(E_{\text{sys}})/\partial\beta_k$
$k = 1$	2.0	0.8471	0.3018	−0.0324
$k = 2$	1.5	0.9359	0.4299	−0.0811
$k = 3$	1.0	1.3833	0.3753	−0.2085

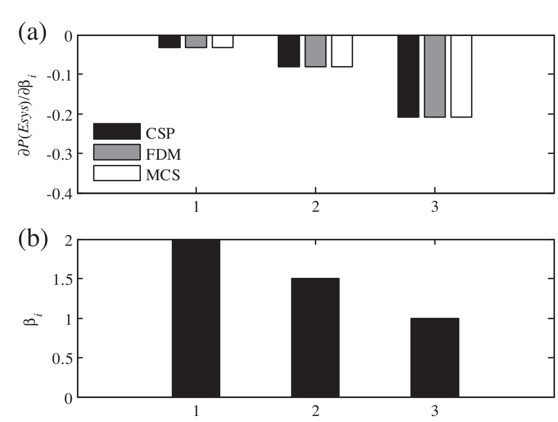

Fig. 1. Series system consisting of 3 components: (a) sensitivities calculated by the CSP method, FDM and MCS, and (b) unequal reliability indices.

Table 2
Sensitivities computed by FDM employing SCM and Monte Carlo simulations (Example 4.1).

$\Delta\beta_k$	FDM: $\partial P(E_{sys})/\partial\beta_k$			MCS: $\partial P(E_{sys})/\partial\beta_k$		
	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
1.0×10^{-1}	−0.0293	-0.0744	−0.1973	-0.0293	−0.0744	−0.1974
1.0×10^{-2}	−0.0324	-0.0807	−0.2076	-0.0324	−0.0807	−0.2076
1.0×10^{-3}	−0.0327	-0.0813	−0.2087	-0.0330	−0.0814	−0.2083
1.0×10^{-4}	−0.0327	-0.0814	−0.2088	-0.0335	−0.0785	−0.2105
1.0×10^{-5}	−0.0327	-0.0814	−0.2088	-0.0323	−0.0784	−0.2161
1.0×10^{-6}	−0.0327	-0.0814	−0.2088	-0.0310	−0.0800	−0.2140
1.0×10^{-7}	−0.0327	-0.0814	−0.2088	-0.0200	−0.1100	−0.2200
1.0×10^{-8}	−0.0327	-0.0814	−0.2088	0.0000	−0.1000	−0.1000
1.0×10^{-9}	−0.0327	-0.0814	−0.2088	0.0000	0.0000	0.0000
1.0×10^{-10}	−0.0327	-0.0814	−0.2088	0.0000	0.0000	0.0000
1.0×10^{-11}	−0.0327	-0.0814	−0.2088	0.0000	0.0000	0.0000
1.0×10^{-12}	−0.0326	-0.0813	−0.2087	0.0000	0.0000	0.0000
1.0×10^{-13}	−0.0322	-0.0799	−0.2087	0.0000	0.0000	0.0000
1.0×10^{-14}	−0.0333	-0.0777	−0.2109	0.0000	0.0000	0.0000
1.0×10^{-15}	0.1110	-0.1110	−0.2220	0.0000	0.0000	0.0000
1.0×10^{-16}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The reliability indices and correlation coefficients are given as

$$\boldsymbol{\beta}=\begin{bmatrix}1.0\\1.0\\2.0\\2.0\\1.5\\1.5\end{bmatrix},\ \mathbf{R}=\begin{bmatrix}1&0.24&0.12&0.06&0.03&0\\0.24&1&0.24&0.12&0.06&0.03\\0.12&0.24&1&0.24&0.12&0.06\\0.06&0.12&0.24&1&0.24&0.12\\0.03&0.06&0.12&0.24&1&0.24\\0&0.03&0.06&0.12&0.24&1\end{bmatrix} \tag{24}$$

Suppose that the sensitivity of the system probability with respect to the reliability index of the sixth component E_6 is of interest. Because E_6 belongs to the second cut-set C_2, $\partial P(E_{C_2})/\partial\beta_6$, $\partial P(E_{sys})/\partial\beta_{C_2}$, and β_{C_2} are computed using the CSP methods for parallel systems and series systems, and the SCM respectively, and substituted to the chain rule formulation in Eq. (20). Fig. 2 illustrates the procedure for the sensitivity calculation of the cut-set system. First, all components except the component E_6 of interest in the second cut-set are compounded. Therefore, an equivalent component E_A is found by compounding the subsystem E_4E_5 and determining its reliability index β_A=2.644. The updated system definition and the correlation coefficient matrix are given as follows

$$E_{sys}=E_{C_1}\cup E_{C_2}=(E_1E_2E_3)\cup(E_AE_6) \tag{25}$$

$$\mathbf{R}=\begin{bmatrix}1&0.24&0.12&0.067&0\\&1&0.24&0.113&0.03\\&&1&0.226&0.06\\&\text{sym.}&&1&0.204\\&&&&1\end{bmatrix} \tag{26}$$

Next, the sensitivity of the second cut-set system failure probability with respect to the reliability index of interest, $\partial P(E_{C_2})/\partial\beta_6$ is computed as −0.001038 using Eq. (18). Compounding the components in the cut-sets, the reliability indices of super-components E_D and E_B are obtained as β_D=2.827 and β_B=3.195, respectively. The updated system definition and its correlation coefficient matrix are given as follows:

$$E_{sys}=E_{C_1}\cup E_{C_2}=E_D\cup E_B \tag{27}$$

$$\mathbf{R}=\begin{bmatrix}1&0.1757\\0.1757&1\end{bmatrix} \tag{28}$$

Using Eq. (15), the sensitivity of the system failure probability with respect to the reliability index of the super-component representing the second cut-set, $\partial P(E_{sys})/\partial\beta_{C_2}$, is computed as −0.002398. Finally, using Eq. (20), the sensitivity is calculated as

$$\begin{aligned}\frac{\partial P(E_{sys})}{\partial\beta_6}&=-\frac{1}{\varphi(-\beta_{C_2})}\cdot\frac{\partial P(E_{sys})}{\partial\beta_{C_2}}\cdot\frac{\partial P(E_{C_2})}{\partial\beta_6}\\&=-\frac{1}{0.002423}(0.002398\times0.001038)\\&=-0.001026\end{aligned} \tag{29}$$

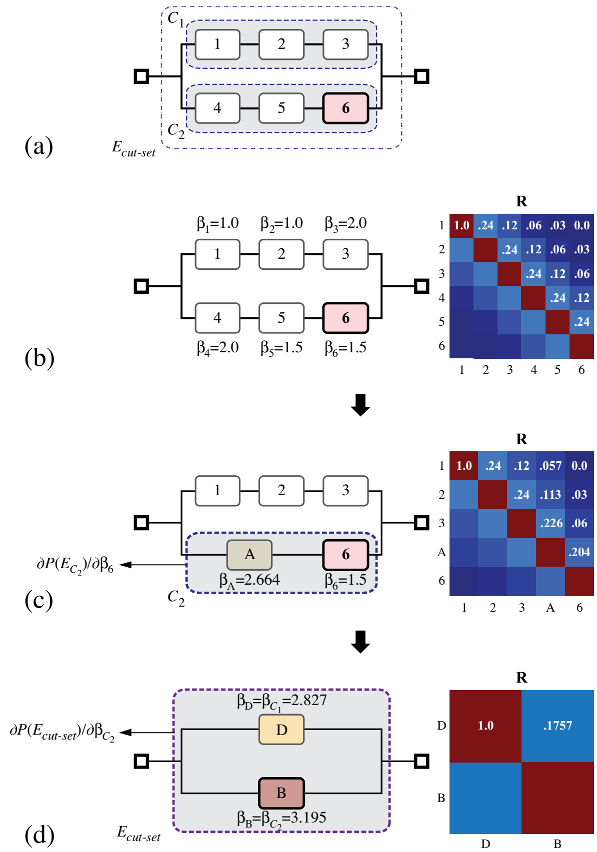

Fig. 2. Illustration of the CSP method to compute sensitivity for a cut-set system with six components: (a) system definition, (b) reliability indices and correlation coefficient matrix, (c) compound component A, updated correlation coefficient matrix, and sensitivity calculation of the second cut-set system, and (d) sensitivity calculation of the system probability with respect to the reliability index of the second cut-set.

The sensitivities of the system failure probability with respect to the other components are shown in Table 3. The results are verified by the FDM with a perturbation of $\Delta\beta_i=10^{-8}$, $i=1,\ldots,6$ and MCS. Comparison results are provided in Table 3 and are shown in Fig. 3. Effects of varying perturbations from 10^{-1} to 10^{-16} in FDM and MCS on sensitivity results were tabulated in Table 4.

Table 3
CSP Sensitivity of the cut-set system failure probability in Example 4.2.

Component	$\partial P(E_{C_1})/\partial\beta_k$	$\partial P(E_{C_2})/\partial\beta_k$	$\partial P(E_{sys})/\partial\beta_{C_j}$	$1/\varphi(-\beta_{C_j})$	$\partial P(E_{sys})/\partial\beta_k$
$k=1$	−0.002701		−0.007317	136.25	−0.002692
$k=2$	−0.002205				−0.002198
$k=3$	−0.004804				−0.004789
$k=4$		−0.001359	−0.002398	412.41	−0.001345
$k=5$		−0.000882			−0.000873
$k=6$		−0.001038			−0.001026

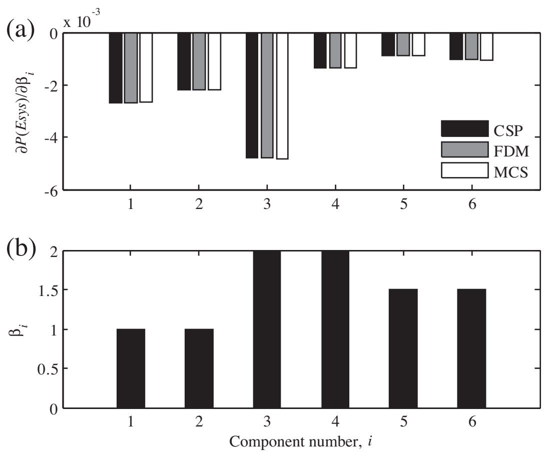

Fig. 3. Cut-set system with two cut-sets and six components: (a) sensitivities computed by the CSP method, FDM and MCS, and (b) unequal reliability indices.

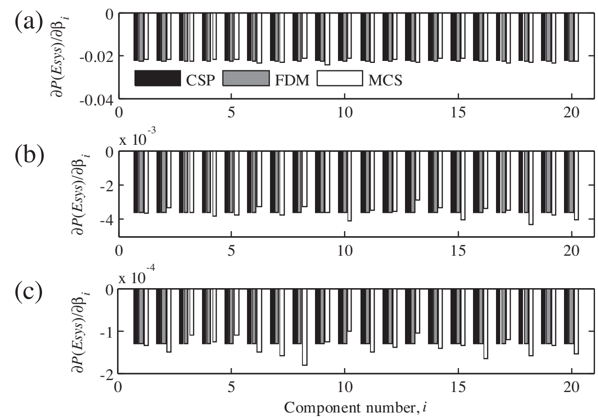

Fig. 4. Sensitivities of series system with 20 components (equal reliability indices and equal correlation coefficients) computed by the CSP method, FDM and MCS: (a) $\beta = 1$; (b) $\beta = 2$; and (c) $\beta = 3$.

4.3. Test example: series system consisting of 20 components with equal reliability indices and equal correlation coefficients

The CSP method is tested for a series system consisting of 20 components that have equal reliability indices, i.e., $\beta_i = \beta_{\text{constant}},\ i = 1, \ldots, 20$ and equal correlation coefficients $\rho_{i,j} = 0.3,\ i \neq j$. As shown in Fig. 4, the results by the CSP method are successfully verified by FDM with a perturbation of $\Delta\beta = 10^{-8}$ and MCS.

4.4. Test example: series system consisting of 20 components with unequal reliability indices and equal correlation coefficients

Suppose that a series system has 20 components having equal correlation coefficients $\rho_{i,j} = 0.3,\ i \neq j$ and their reliability indices are given by

$$\beta_i = 2 \cdot \left(1 - \frac{i-1}{19}\right),\ i = 1, \ldots, 20 \qquad (30)$$

Table 4
Finite difference sensitivity and Monte Carlo simulation sensitivity computation in Example 4.2.

$\Delta\beta_k$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
FDM: $\partial P(E_{sys})/\partial\beta_k$						
1.0×10^{-1}	−0.002641	−0.002182	−0.004434	−0.001254	−0.000847	−0.000983
1.0×10^{-2}	−0.002703	−0.002195	−0.004767	−0.001345	−0.000870	−0.001027
1.0×10^{-3}	−0.002709	−0.002195	−0.004802	−0.001353	−0.000870	−0.001023
1.0×10^{-4}	−0.002709	−0.002195	−0.004805	−0.001354	−0.000870	−0.001023
1.0×10^{-5}	−0.002709	−0.002195	−0.004806	−0.001354	−0.000870	−0.001023
1.0×10^{-6}	−0.002709	−0.002195	−0.004806	−0.001354	−0.000870	−0.001023
1.0×10^{-7}	−0.002709	−0.002195	−0.004806	−0.001354	−0.000870	−0.001023
1.0×10^{-8}	−0.002709	−0.002195	−0.004806	−0.001354	−0.000870	−0.001023
1.0×10^{-9}	−0.002709	−0.002195	−0.004806	−0.001354	−0.000870	−0.001023
1.0×10^{-10}	−0.002708	−0.002195	−0.004805	−0.001353	−0.000869	−0.001023
1.0×10^{-11}	−0.002709	−0.002188	−0.004807	−0.001354	−0.000866	−0.001021
1.0×10^{-12}	−0.002668	−0.002223	−0.004774	−0.001332	−0.000890	−0.001001
1.0×10^{-13}	−0.002246	−0.002246	−0.004454	−0.001106	0.000000	−0.001106
1.0×10^{-14}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-15}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-16}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
MCS: $\partial P(E_{sys})/\partial\beta_k$						
1.0×10^{-1}	−0.002641	−0.002175	−0.004450	−0.001257	−0.000846	−0.000980
1.0×10^{-2}	−0.002706	−0.002180	−0.004719	−0.001327	−0.000885	−0.001014
1.0×10^{-3}	−0.002708	−0.002276	−0.004724	−0.001356	−0.000916	−0.001034
1.0×10^{-4}	−0.003120	−0.002220	−0.004520	−0.001220	−0.000960	−0.000860
1.0×10^{-5}	−0.002720	−0.002120	−0.004680	−0.001390	−0.000830	−0.000880
1.0×10^{-6}	−0.002000	−0.002000	−0.010000	0.000000	0.000000	0.000000
1.0×10^{-7}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-8}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-9}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-10}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-11}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-12}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-13}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-14}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-15}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.0×10^{-16}	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

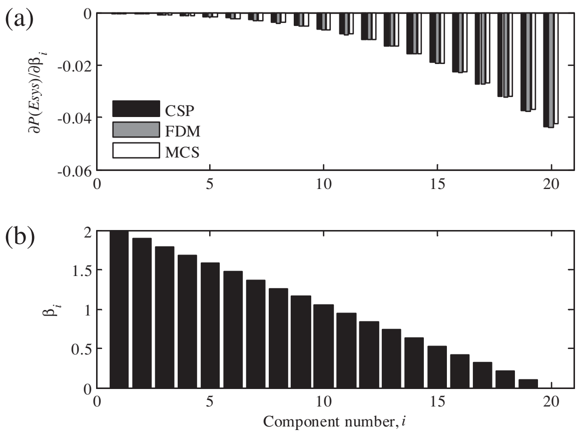

Fig. 5. Sensitivities of series system with 20 components (unequal reliability indices and equal correlation coefficients): (a) comparison between the CSP method, FDM and MCS, and (b) component reliability indices.

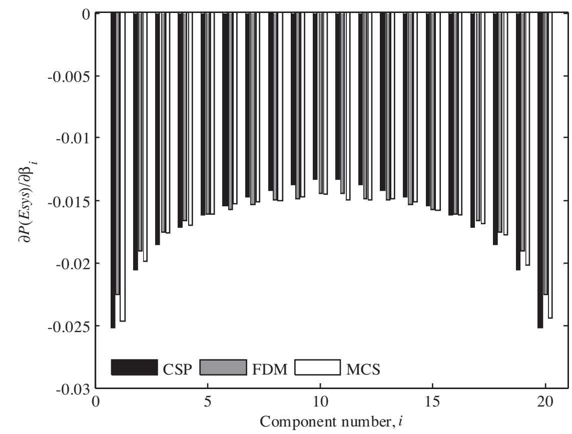

Fig. 7. Sensitivities of series system with 20 components (equal reliability indices $\beta = 2.0$ and unequal correlation coefficients in Fig. 6): comparison between the CSP method, FDM and MCS.

Fig. 5 compares the results from the CSP method with those from FDM with perturbation of $\Delta\beta = 10^{-8}$ and MCS. It is noted that the system failure probability is most sensitive to β_{20} because the 20-th component contributes most to the system failure probability.

4.5. Test example: series system consisting of 20 components with equal reliability indices and unequal correlation coefficients

Consider a series system problem consisting of 20 components having equal reliability indices and unequal correlation coefficients determined by

$$\rho_{i,j} = 1 - \sqrt{\frac{|i-j|}{19}}, \quad i,j = 1,\ldots,20 \tag{31}$$

The distribution of the correlation coefficients in the matrix is visualized in Fig. 6. The equal reliability indices of the components are used, i.e., $\beta_i = 2,\ i = 1,\ldots,20$. Fig. 7 compares the results of sensitivity calculations by the CSP method, FDM with a perturbation of $\Delta\beta = 10^{-8}$ and MCS, which show symmetry due to the correlation structure.

4.6. Test example: series system consisting of 20 components with randomly generated reliability indices and correlation coefficients

The CSP method is tested for a series system consisting of 20 components with randomly generated unequal reliability indices (Table 5), and correlation coefficients (Fig. 8). The results from the CSP method are verified by FDM with varying perturbations of $\Delta\beta = 10^{-3}$, $\Delta\beta = 10^{-8}$ and $\Delta\beta = 10^{-9}$ and MCS. It is noted that sensitivity calculations by FDM may result in large error or even different signs depending on perturbation values, as shown in Fig. 9. This is one of the motivations for the development of CSP method, which calculates sensitivity efficiently and accurately.

Table 5
Reliability indices in Example 4.6.

Component, i	β_i	Component, i	β_i
1	2.28384	11	1.45275
2	1.59273	12	2.14048
3	1.72373	13	2.16704
4	1.49318	14	1.75261
5	1.57674	15	2.88052
6	2.43578	16	1.78762
7	1.81745	17	1.89771
8	2.01840	18	1.90212
9	2.02029	19	2.51031
10	2.10169	20	1.15859

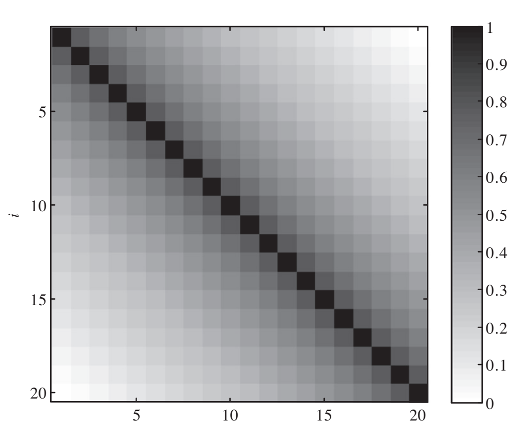

Fig. 6. Correlation coefficient matrix, $\rho_{i,j} = 1 - \sqrt{|i-j|/19},\ i,j = 1,\ldots,20$.

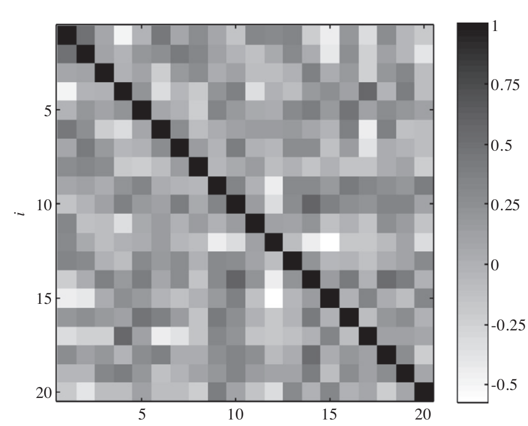

Fig. 8. Randomly generated correlation coefficient matrix, $\rho_{i,j}$ used in Example 4.6 and Example 4.7.

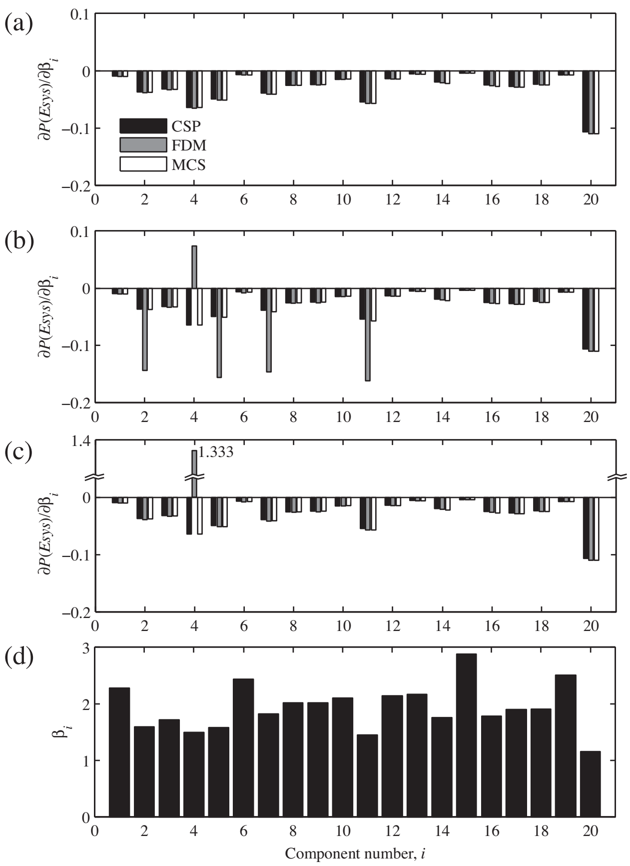

Fig. 9. Sensitivities of parallel system with 20 components (random reliability indices and unequal correlation coefficients): comparison between the CSP method, FDM and MCS: (a) FDM perturbation, $\Delta\beta=10^{-3}$, (b) FDM perturbation, $\Delta\beta=10^{-8}$, (c) FDM perturbation, $\Delta\beta=10^{-9}$, and (d) reliability indices.

4.7. Test example: parallel system consisting of 20 components with randomly generated reliability indices

A parallel system having 20 components with randomly generated reliability indices is tested. For the testing purposes, two correlation coefficient matrices are used: a randomly generated one (Fig. 8), and the equal correlation coefficient (0.5). Fig. 10(a) and (b) show comparison results from randomly generated reliability indices in Fig. 10(c). The randomly generated reliability indices are listed in Table 6. To test the method for cases in which components have both positive and negative signs, another set of reliability indices are randomly generated as shown in Fig. 10(e) and Table 6. Sensitivities are calculated with equal correlation coefficients (0.5) as shown in Fig. 10(d). Overall, the CSP method shows good agreements.

4.8. Test example: parallel system consisting of 65 components with equal reliability indices and unequal correlation coefficients

To test the CSP method for parallel systems and also for systems with larger number of components, consider a parallel system consisting of 65 components having equal reliability indices $\beta_i = -1.5,\ i = 1, \ldots, 65$. The correlation coefficients are given by

$$\rho_{i,j} = 1 - \sqrt{\frac{|i-j|}{64}},\ i,j = 1, \ldots, 65 \qquad (32)$$

The results in Fig. 11 show relatively larger differences among the CSP method, FDM with perturbation of $\Delta\beta = 10^{-8}$ and MCS (10^6

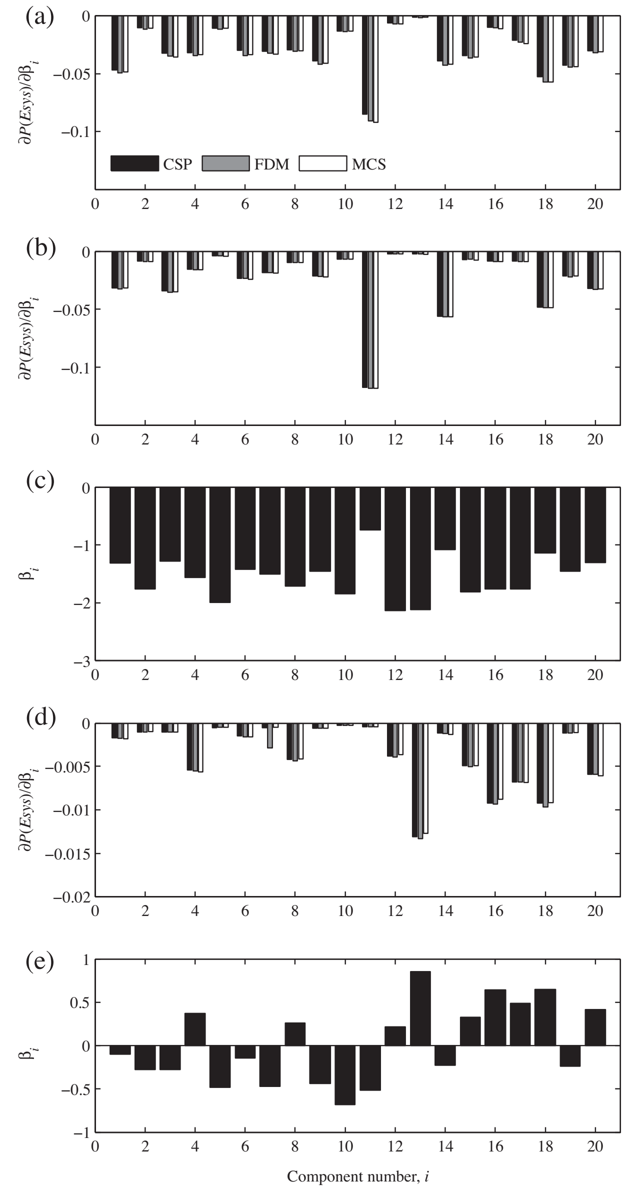

Fig. 10. Sensitivities of parallel system with 20 components with random reliability indices: (a) sensitivities with the correlation coefficient matrix in the test Example 4.6, (b) sensitivities with equal correlation coefficients $\rho_{i,j} = 0.5$, (c) randomly generated reliability indices, (d) sensitivities with equal correlations $\rho_{i,j} = 0.5$, and (e) randomly generated reliability indices.

Table 6
Reliability indices in Example 4.7.

Component, *i*	Fig. 10(c) β_i	Fig. 10(e) β_i	Component, *i*	Fig. 10(c) β_i	Fig. 10(e) β_i
1	−1.30407	−0.09583	11	−0.73897	−0.51614
2	−1.75792	−0.27721	12	−2.13813	0.21825
3	−1.27964	−0.27396	13	−2.11614	0.85414
4	−1.56172	0.37304	14	−1.07950	−0.23074
5	−1.99004	−0.48342	15	−1.80698	0.33008
6	−1.41823	−0.14431	16	−1.75581	0.64524
7	−1.50297	−0.47295	17	−1.75522	0.48637
8	−1.70968	0.25786	18	−1.14115	0.64556
9	−1.45271	−0.43553	19	−1.45254	−0.23998
10	−1.83899	−0.67939	20	−1.30119	0.41603

samples) than the previous examples. It should be noted that the probability of the parallel system keeps decreasing as the

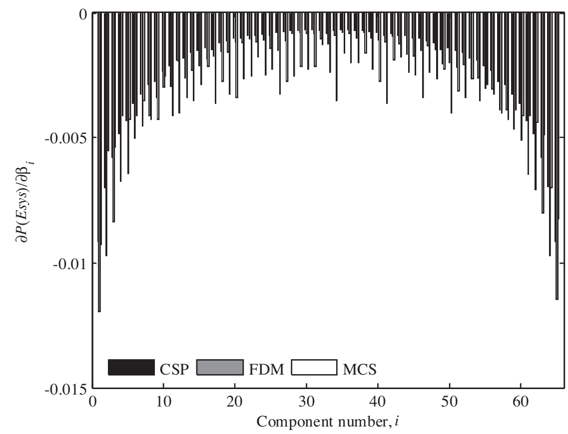

Fig. 11. Sensitivities of parallel system with 65 components (equal reliability indices $\beta = -1.5$ and unequal correlation coefficients): comparison between the CSP method, FDM and MCS.

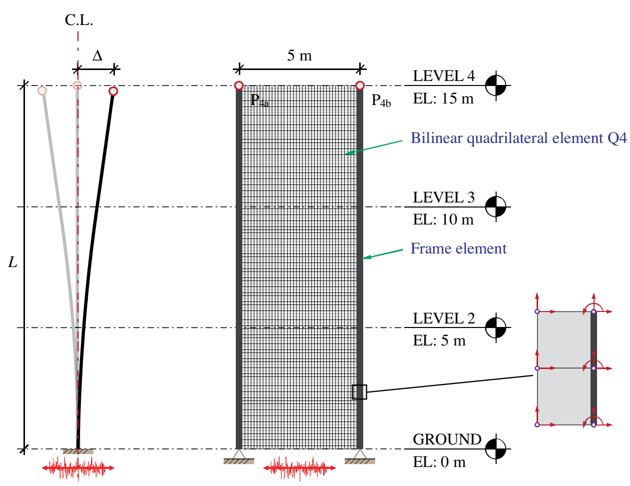

Fig. 12. Structure geometry and loading configuration for the first-passage probability application example.

compounding continues, which may make the parameter sensitivity calculations more sensitive to the numerical errors for all three approaches.

4.9. Application example: sensitivity of the first passage probability

The CSP method is applied to calculate the sensitivity of the first passage probability. Consider a building subjected to a stochastic earthquake ground motion as shown in Fig. 12. A discrete representation method [25] can be used to model the continuous stochastic process by use of a finite number of standard normal random variables. In this example, the stochastic process of the ground acceleration, $f(t)$, is modeled as a filtered Gaussian process using the discrete representation method as follows:

$$f(t) \cong \sum_{i=1}^{n} \sqrt{2\pi\Phi_0/\Delta t} \cdot v_i \cdot h_f(t - t_i)\Delta t = s(t)^{\mathrm{T}}\mathbf{v} \tag{33}$$

where Φ_0 is the constant power spectral density (PSD) of the underlying white noise that enters the filter, Δt is the time step of the discretization, $\mathbf{v} = [v_1, v_2 \ldots, v_n]^{\mathrm{T}}$ is the vector of n uncorrelated standard normal random variables, $h_f(\cdot)$ denotes the impulse-response function describing the filter, $t_i, i = 1, \ldots, n$ are the discretized time points, and $\mathbf{s}(t)$ is the vector of the deterministic functions that describe the filter characteristics and the intensity of the input process. The displacement time history $u(t)$ of a linear structure subjected to the stochastic excitation $f(t)$ is derived as

$$u(t) = \int_0^t f(\tau)h_s(t-\tau)d\tau = \int_0^t \sum_{i=1}^{n} v_i s_i(\tau)h_s(t-\tau)d\tau$$
$$= \sum_{i=1}^{n} v_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}}\mathbf{v} \tag{34}$$

where $h_s(t)$ is the unit impulse response function of the structural response of interest and $\mathbf{a}(t)$ is the vector of deterministic function describing the filter characteristics, intensity of the input process and the dynamic characteristics of the structure. In this example, the stochastic seismic excitation is modeled as a filtered white-noise process using the Kanai–Tajimi filter model [26]. Its unit-impulse response function [13] and the power spectral density (PSD) function are given as

$$h_f(t) = \exp(-\zeta_f\omega_f t)\left[\frac{(2\zeta_f^2 - 1)\omega_f}{\sqrt{1-\zeta_f^2}} \sin\left(\omega_f\sqrt{1-\zeta_f^2}\cdot t\right) - 2\zeta_f\omega_f \cos\left(\omega_f\sqrt{1-\zeta_f^2}\cdot t\right)\right] \tag{35}$$

$$\Phi(\omega) = \frac{1 + 4\zeta_f^2(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2 + (2\zeta_f\omega/\omega_g)^2}\Phi_o \tag{36}$$

where ω_f and ζ_f are the filter parameters representing the predominant frequency and the bandwidth of the process respectively.

For the dynamic finite element analysis, continuum elements in the building model are modeled by standard quadrilateral elements (see Fig. 12), however, other types of elements could also be used – see, for example, reference [27]. The structural columns represented by the two vertical lines are modeled with frame elements. Young's modulus E = 21,000 MPa and density ρ = 2400 kg/m^3 are used as material properties for both the quadrilateral and the frame elements. The damping matrix is constructed using a Rayleigh damping model such as

$$\mathbf{C} = \kappa_0\mathbf{M} + \kappa_1\mathbf{K} \tag{37}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structure, respectively. The coefficients κ_0=2.34 and κ_1=1.18×10^{-4} are used for the Rayleigh damping model to achieve 2% damping. Table 7 shows the Kanai–Tajimi filter parameters, column size, time interval of interest, time step for the dynamic finite element analysis as well as the threshold value of the average drift ratio at each time point.

The component failure event is defined at each time point to describe the event that the average of the inter-story drift ratios computed at the marked (red) points in Fig. 12 exceeds the given threshold value, i.e.,

$$u_0 - \left(\frac{u(t_i)_{\mathrm{Left}} + u(t_i)_{\mathrm{Right}}}{2L}\right) = u_0 - \left(\frac{(\mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{\mathrm{Left}} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{\mathrm{Right}})\mathbf{v}}{2}\right) \leqslant 0 \tag{38}$$

Table 7
One-story building example: parameters used for design domain, probabilistic constraint and ground motion model.

Φ_o	ω_f	ζ_f	Column size	Thickness	t_{interval}	Δt	u_0
			m	m	s	s	
500	5π	0.4	0.4 × 0.4	0.1	5.0	0.04	0.02

where u_0 is the threshold value on the inter-story drift ratio, $u(t_i)_{\text{Left}}$ and $u(t_i)_{\text{Right}}$ are respectively the displacement at the red points in Fig. 12 at time $t = t_i$, and L is 15 m, and $\tilde{\boldsymbol{\rho}}$ is the vector of the design variables describing the material density distribution in the continuum elements. As explained above, the first-passage probability can be computed by obtaining the probability of the series system consisting of the component events described in Eq. (38) defined at 125 discrete time points. The reliability index of each component failure event is computed from $\beta(u_0, t_i) = u_0 / \|\mathbf{a}(t_i)\|$ [25]. The correlation coefficients are obtained by the inner-product of the negative normalized gradient vectors at the so-called design point or most probable point [23,25]. Fig. 13 shows the correlation coefficient matrix for the component failure events of this example. Fig. 14(a) and (b) show one of the input excitation time histories that could be randomly generated from the Kanai–Tajimi filter model described above, and the corresponding time history of the average inter-story drift ratio at the red points. Fig. 14(c) and (d) respectively show the reliability indices at the discretized time points, and the first passage probabilities up to each time point.

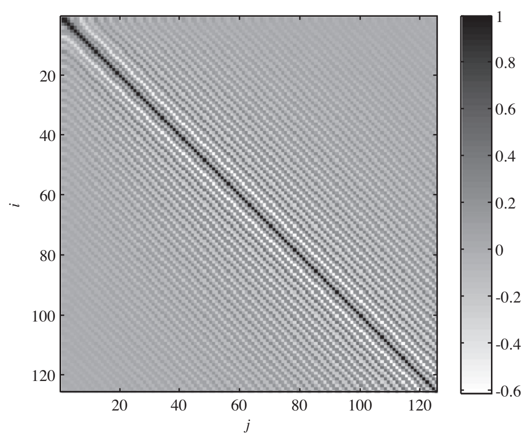

Fig. 13. Correlation coefficient matrix between component failure events for the first-passage probability application example.

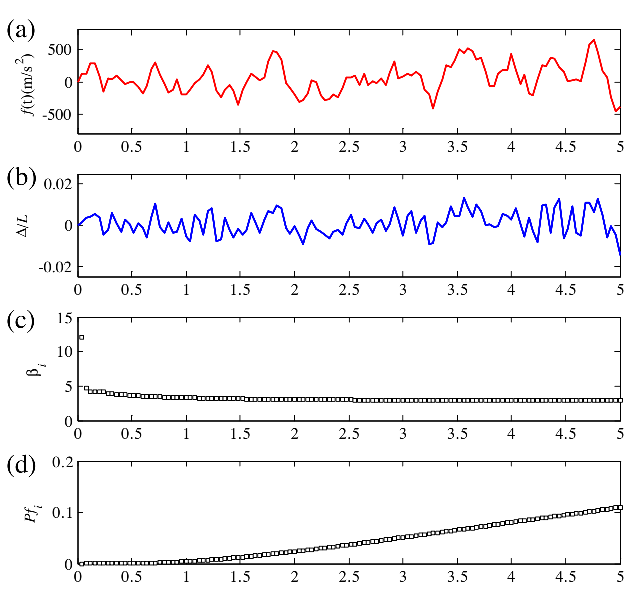

Fig. 14. First-passage probability example: (a) a randomly generated excitation, (b) corresponding dynamic responses of story drift ratio, (c) reliability index (at each time instance), and (d) first passage probability (up to each time instance).

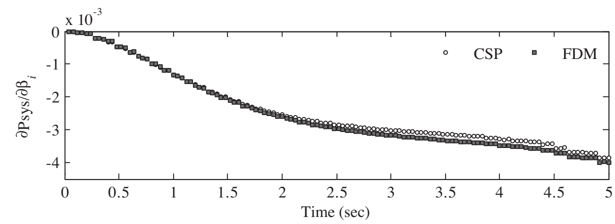

Fig. 15. Sensitivity of the first-passage probability with respect to the reliability indices at discrete time points.

Fig. 15 shows the sensitivity of the first-passage probability for the 5-second period with respect to the reliability indices at the discretized points computed by the CSP method. The results are successfully verified by use of the results from the finite difference method with a perturbation of $\Delta\beta = 10^{-5}$. The calculated parameter sensitivities are useful for design or topology optimization under the constraint on the first-passage probability of a structural system [7,12,16].

5. Concluding remarks and extensions

In this paper, a method is developed to compute the parameter sensitivity of series, parallel and general system problems using the sequential compounding method (SCM). For series or parallel systems, the proposed Chun–Song–Paulino (CSP) method obtains the parameter sensitivity using the intermediate results of the second last sequential compounding. For general systems such as cut-set or link-set problems, the CSP method for series and parallel systems are used at the cut-set level or system-level to compute the sensitivity of interest. By means of a wide range of numerical examples, the accuracy of the CSP method was successfully demonstrated for series, parallel and cut-set systems under various conditions on the component reliability indices and correlation coefficients. The accuracy of the sensitivities calculated for parallel systems seems to be more sensitive to numerical errors than other types of system events. The method was successfully applied to compute the sensitivities of the first passage probability of the structure subjected to a stochastic ground motion. The CSP method is expected to facilitate efficient use of gradient-based optimization algorithms for design or topology optimization under constraints on system failure probability including the first-passage probability. When the sensitivity is computed with respect to each of the component events, the computational efficiency can be further improved in future research by recycling intermediate results of sequential compoundings. It is also noted that compounding orders may affect the probability and sensitivity computed by the sequential compounding method especially when component events are highly dependent. In order to address this issue, optimal orders of compounding procedures or general ordering schemes need to be explored in future research.

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