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Shortcuts to flexible structures

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While this Commentary "paper" will be distributed primarily electronically, some of us will choose to read it on physical paper. For many centuries, the flexibility, stability, strength, and extreme thinness of paper have made it an essential medium of information exchange, one that has shown surprising endurance into the digital age. These same properties have also rendered it invaluable as a structural material, from the nests of paper wasps to modern sustainable containers for commercial products. Despite paper's long familiarity, its versatility continues to yield novel applications, as in the new study by Dang, Gonella, and Paulino (1). The authors combine cutting, folding, and gluing to direct paper's strength and flexibility in new ways, giving rise to a new form of flexible structure they name folded kirigami.

Folded kirigami is formed from a single sheet of paper, into which a repeating pattern of cuts and fold lines is placed. From this two-dimensional configuration, the panels are folded along the lines to create a structure of strong, rigid but hollow paper prisms joined via flexible paper hinges. Strikingly, these three-dimensional structures are each formed from a single sheet of paper.

These structures differ from conventional solids, which have fixed shapes that can only be deformed via substantial force. A flexible solid structure might be defined as one containing elements of fixed shape that are allowed to move relative to one another, such as the twin blades of scissors, or the bones of the human body. Such structures are relatively rare in the physical universe and may have first proliferated in the Earth's seas with primitive string-like biomolecules that had fixed backbones that could twist freely into many conformations. Flexibility has since become predominant in the biological world: Single-cell organisms squeeze through gaps, leaves unfurl to capture solar energy, muscles contract, and feathers reposition to soar in the wind.

Flexibility emerges, then, not because it is easy to achieve but because it is useful. This principle drives engineered flexible structures, from the aforementioned scissors to sophisticated robots, and from solar panels that, once unfolded, power orbital space missions to stents inserted into the human heart.

Folded kirigami is a type of mechanical metamaterial, a recent and powerful paradigm for the production of flexible solids: engineered structures that have repeated geometric motifs (generally periodic unit cells) that imbue them with novel mechanical response (2, 3). Flexible mechanical metamaterials combine simple structure with rich function, and thus hold great promise in arenas inaccessible to traditional manufacturing techniques, such as nanotechnology.

There are a number of existing techniques to produce metamaterials and other structures at the smallest scales. Lithography consists of removing material (cutting it away) to achieve a target structure. Additive manufacturing (3D printing) can build up material with nanoscale features. Self-assembly can connect many individual building blocks into much larger structures. However, all of these techniques have certain limitations: Self-assembly can be prone to defects and often forms relatively weak bonds; other techniques don't always scale well. In general, flexible structures represent a particular challenge for experimental realization. Because flexible structures can exist in any of several configurations nearly degenerate in energy, they can actually deform during their own creation, and constructing an object while it is changing shape is difficult. This is one reason that naturally occurring atomic structures such as crystals are almost never flexible [one notable exception is zeolites, which have a "flexibility window" that endows them with unusual properties (4)].

Flexibility can be understood as a sort of selectivity, the ability to deform more easily into some shapes than into others. Conventional solids are not selective: Deformation into any shape requires substantial energy. Nor are conventional fluids: They can take any shape (at least those preserving volume, in the case of liquids) without the input of energy. In contrast, a thin sheet—whether it consists of paper or more specialized materials—selects for bending over stretching because the energetic cost of stretching is linear in the thickness, whereas the cost of bending is cubic in the thickness. As such, paper has a strong tendency to bend rather than to stretch. However, many flexible metamaterials are based around mechanisms, in which rigid elements such as one-dimensional rods, two-dimensional plates, or three-dimensional polyhedra are joined via flexible hinges such that they may transform their shape by bending the hinges without deforming the rigid elements. The selectivity of the design depends, then, on the ability to achieve highly flexible hinges with highly rigid elements. In the ideal case, only the hinges deform and the other elements remain rigid, selecting entirely for the desired deformation. In the opposite limit, the so-called rigid elements deform as easily as the hinges.

One way to address this challenge of selectivity is by using multiple materials and hand-assembling a structure. However, this procedure does not readily scale, and previous techniques have not been able to achieve high selectivity with a structure formed from a single sheet. Consider, in

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particular, removing material from a sheet, whether it consists of paper or single-atomic layer graphene, to generate a network of solid rectangles joined via thin ligaments (5). In an ideal world, this generates a mechanism, in which the elements counterrotate in order to achieve dilation in the plane rather than bending. In practice, the structure buckles into the third dimension, and the thinner the ligaments, the more quickly they age and tear. Indeed, crumpling into the third dimension radically alters graphene's bending stiffness, giving it a dimensionless bending stiffness comparable to paper despite its vastly thinner aspect ratio (when the crumpling itself is ignored). While such buckling is desirable for certain applications, it is contrary to the vision of two-dimensional deformation discussed below.

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The goal, then, is to create a ligament that is at once large and robust while also being thin and flexible. The authors confront this paradox by cleaving to the third dimension rather than fleeing it. The authors cut slits in their sheets but do not in general remove the material. Instead, the material is folded along machined creases to form rigid prisms (which are then rendered permanent via the manual application of an adhesive). The result is a network of hollow paper prisms with polygonal cross-sections joined at their corners via vertical creases in a single piece of paper. The crease can be quite lengthy, then, while appearing as nearly a geometric point in the plane of the material.

This elegant approach achieves a high degree of selectivity, combining durable, flexible hinges, and stiff, lightweight rigid elements. As such, it presents an advanced platform to realize certain phenomena that have lagged behind theoretical predictions. One such phenomenon is the Kane Lubensky topological polarization (6), in which certain mechanical structures with similar numbers of constraints and degrees of freedom (isostaticity, in essence) such as a network of corner-sharing triangles possess topological modes on one boundary but not on the other. The minimum stiffness of the surface with topological modes is given by the stiffness of the flexible hinges, whereas the maximum stiffness of the opposing surface without any such modes is given by the stiffness of the rigid elements (7). Hence, selectivity is a key parameter in realizing the benefits of topological polarization. The present work demonstrates that even in fairly small systems, the surface stiffnesses of the two sides differ by a factor of five (figure 4E), a dramatic improvement over previous state-of-the-art realizations via 3D printing (8). Further improvements may require incorporating more unit cells (and hence using larger sheets): a deformation that varies over ten surface cells may require 100 cells to decay into the bulk (9).

One of the appeals of flexibility is the ability to achieve nonlinear behavior under relatively gentle loads. A hallmark of linear (and energy-conserving) systems is reciprocity, which implies that if a source at one point generates a signal at a second, then a source at the second point will also generate a signal at the first. This is the reason that true one-way glass is impossible, at least with linear optics. In mechanics, though, nonlinear elements are common, and nonreciprocity has been demonstrated in metamaterials (10). In the present work, the high flexibility of the structures allows angles to change by large amounts, leading to geometric nonlinearities (just as the displacement of a pendulum is nonlinear in the rotation angle). Despite the fact that rigid panels are deformed only slightly, these geometric nonlinearities induce nonreciprocity (by a factor of approximately two) under light loading conditions ($\approx$0.2 N).

While these phenomena are rather general, in practice they are best realized in structures whose deformations are confined to two dimensions. One reason is that two-dimensional critically coordinated (isostatic) structures have a Guest Hutchinson mode (11, 12) which forms a natural mechanism, the equivalent result in three dimensions is the presence of three such modes, leading to more difficult dynamics and less stable structures (ones that in fact may not be able to support their own weight, as flexibility is increased). Another sort of mechanical criticality also applies to any two-dimensional mechanism-based structure, in which the global mechanism mode can be promoted to a mechanism field that generates deformations governed by a geometric compatibility condition (13, 14). In the latter case, the selectivity of folded kirigami may drastically improve the effectiveness of the theory, as it already has in the former case.

Despite their success in realizing these quasi-two-dimensional deformations, folded kirigami structures may also have application to the complexities of the third dimension. Deployable origami tubes have previously been assembled from multiple sheets (15), as have more complex prismatic structures that deform in three dimensions (16). Three-dimensional structures of comparable complexity and flexibility have not, however, been achieved via a single sheet as has been done in the present work.

Complexity can also be achieved in two dimensions. An increasing focus of metamaterials research is on more intricate structures, with more complicated structures than simply repeating a unit cell. Complex response, in which the shape deforms in a certain way in response to a given stimulus, has already been designed via optimization techniques (17, 18). As with many theoretical designs, its practical realization is limited by the selectivity of the flexible hinges. Because of the complicated structures, they may prove more difficult to realize via folded kirigami, but also may benefit more greatly from its hinge structure.

The folding kirigami technique is sure to be adopted by other researchers. As techniques are further refined and extended, they may manage to achieve structures at new length scales (19), or with higher selectivity, or with more self-folding abilities (20), or greater numbers of unit cells. Whatever the ultimate result of such inquiries, the folded kirigami technique is a vital breakthrough that breathes new life into the field of mechanical metamaterials, complementing existing techniques based on additive manufacturing, lithography, and assembly.

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